

a) The total Hilbert space dimension of  $n_1$  spin 1's and  $n_{3/2}$  spin 3/2's is

$$N_s = 3^{n_1} 4^{n_{3/2}} = 2^{n_1 \log 3 + 2 n_{3/2}}$$

where the log is base 2.

The total Hilbert space dimension of  $n$  qubits is

$$N_q = 2^n$$

We need  $N_q \geq N_s$  in order for the qubits to simulate the spins

$$N_q \geq N_s \therefore n \geq n_1 \log 3 + 2 n_{3/2}$$

$$\therefore n = \lceil n_1 \log 3 + 2 n_{3/2} \rceil$$

Here  $\lceil x \rceil$  denotes the ceiling function (round up to nearest integer).

b) Consider each spin separately. Clearly a single spin-1 requires 2 qubits, as does a single spin-3/2. So for a bunch of them

$$n = 2 n_1 + 2 n_{3/2}$$

c) In both cases

$$n = O(n_1) + O(n_{3/2})$$

So they are equally efficient. The first case has better coefficients, though (but I don't ask for that).

d) 12 basis states for  $n_1 = n_{3/2} = 1$ . We need 4 qubits at least (16 basis states). We arbitrarily assign a different qubit basis state to each spin basis state. For example

$$\begin{array}{l}
 | -1 \rangle \otimes | -3/2 \rangle \\
 | -1 \rangle \otimes | -1/2 \rangle \\
 | -1 \rangle \otimes | 1/2 \rangle \\
 | -1 \rangle \otimes | 3/2 \rangle \\
 | 0 \rangle \otimes | -3/2 \rangle \\
 | 0 \rangle \otimes | -1/2 \rangle \\
 | 0 \rangle \otimes | 1/2 \rangle \\
 | 0 \rangle \otimes | 3/2 \rangle \\
 | +1 \rangle \otimes | -3/2 \rangle \\
 | +1 \rangle \otimes | -1/2 \rangle \\
 | +1 \rangle \otimes | 1/2 \rangle \\
 | +1 \rangle \otimes | 3/2 \rangle
 \end{array}
 \longrightarrow
 \begin{array}{l}
 | 0000 \rangle \\
 | 0001 \rangle \\
 | 0010 \rangle \\
 | 0011 \rangle \\
 | 0100 \rangle \\
 | 0101 \rangle \\
 | 0110 \rangle \\
 | 0111 \rangle \\
 | 1000 \rangle \\
 | 1001 \rangle \\
 | 1010 \rangle \\
 | 1011 \rangle
 \end{array}$$