The condition for $f(n) = O(g(n))$ is

$$f(n) < c\, g(n) \quad \forall\, n > n_0$$

which can be restated

$$\frac{f(n)}{c\, g(n)} < 1 \qquad \forall \quad n > n_0 \qquad (1)$$

(a) $f(n) = O(g(n))$ if

$$f(n) \leq C\, g(n) \qquad \forall\, n > n_0$$

for arbitrary C and n_0

$f(n) = \Omega(g(n))$ if

$$f(n) \geq C\, g(n) \qquad \forall\, n > n_0$$

for arbitrary C and n_0.

Note: the C and n_0 need not be the same

Clearly

$$f(n) \leq C\, g(n) \qquad \forall\, n > n_0$$

Implies

$$g(n) \geq \frac{1}{C} f(n) \qquad \forall\, n > n_0$$

So $g(n) = \Omega(f(n))$ if $f(n) = O(g(n))$

(b) $f(n) = \sum_{j=0}^{k} a_j n^j$ $\qquad g(n) = n^l$

$$\therefore \frac{f(n)}{g(n)} = \sum_{j=0}^{k} a_j n^{j-l}$$

If $l > k$ then all n^{j-l} for $j \leq k$ will vanish as $n \to \infty$, so

$$\frac{f(n)}{g(n)} \to 0 \quad \text{as } n \to \infty$$

$\therefore f(n) = o(n^l)$ by Eg. 1

(c) If $\frac{a(n)}{c\,g(n)} < 1 \quad n > n_0$

$$\frac{b(n)}{c'h(n)} < 1 \quad n > n_0'$$

then $\frac{a(n)}{c\,g(n)} \frac{b(n)}{c'h(n)} < 1 \quad n > \max(n_0, n_0')$

$$\therefore \frac{a(n)\,b(n)}{cc'g(n)\,h(n)} < 1 \qquad n > \max(n_0, n_0')$$

$\therefore a(n)\,b(n) = O(g(n)\,h(n))$ if $a(n) = O(g(n))$
$b(n) = O(h(n))$

The rest are just more of the same