1. Prove

$$\prod_{j \in C} P_{s_j, S_j} = \left(\prod_{j \in C \setminus k} P_{s_j', S_j'} \right) P_{s_k', S_k'}$$

Since the projectors commute

$$\prod_{j \in C} P_{s_j, S_j} = \left(\prod_{j \in C \setminus k} P_{s_j, S_j} \right) P_{s_k, S_k}$$

$$= \left(\prod_{j \in C \setminus k} P_{s_j, S_j} P_{s_k, S_k} \right) P_{s_k, S_k}$$

Using $(s_j S_j)^2 = 1 \quad \forall j$

$$P_{s_j, S_j} P_{s_k, S_k} = \frac{1}{4}\left(1 + s_j S_j + s_k S_k + s_j s_k S_j S_k\right)$$

$$P_{s_j', S_j'} P_{s_k, S_k} = \frac{1}{4}\left(1 + s_j s_k S_j S_k + s_k S_k + s_j S_j\right)$$

$$\therefore \quad P_{s_j, S_j} P_{s_k, S_k} = P_{s_j', S_j'} P_{s_k, S_k}$$

$$\therefore \prod_{j \in C} P_{s_j, s_j} = \left(\prod_{j \in C \setminus k} P_{s_j', s_j'} P_{s_k, s_k} \right) P_{s_k, s_k}$$

$$= \left(\prod_{j \in C \setminus k} P_{s_j', s_j'} \right) P_{s_k, s_k}$$

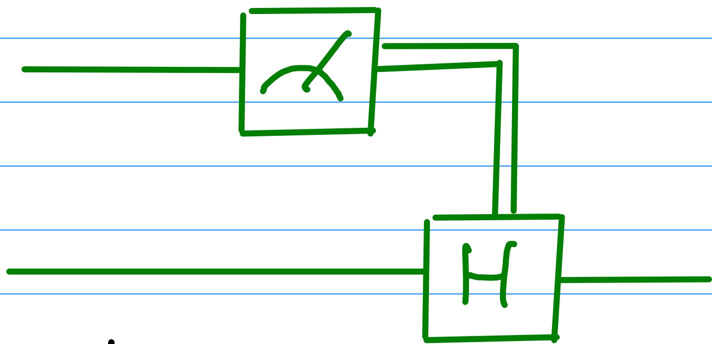
then using

$$P_{s_k, s_k} = P_{s_k', s_k'}$$

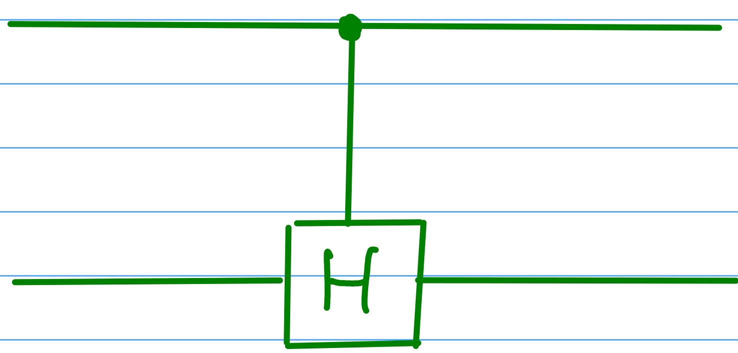
we get the required result

2

A possible Clifford circuit is



If delayed measurement were possible, this would imply we had the gate



$$= |0\rangle\langle 0| \otimes \mathbb{1} + |1\rangle\langle 1| \otimes H = U$$

$$U = \mathbb{1} \otimes \mathbb{1} + \sigma_z \otimes \mathbb{1} + \mathbb{1} \otimes H - \sigma_z \otimes H$$
$$= \mathbb{1} \otimes |H+\rangle\langle H+| + \sigma_z \otimes |H-\rangle\langle H-|$$

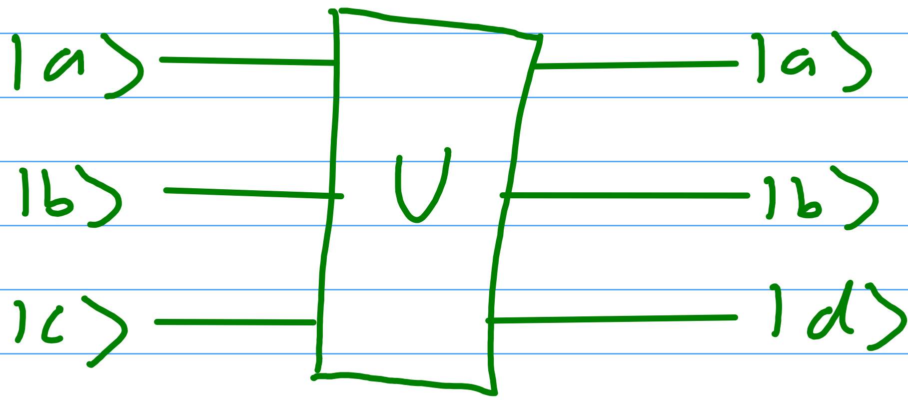
Using $H = |H+\rangle\langle H+| - |H-\rangle\langle H-|$

$$U \, \sigma_x \otimes \mathbb{1} \, U^{+} = \sigma_x \otimes |H+\rangle\langle H+| - \sigma_x \otimes |H-\rangle\langle H-|$$

$$= \sigma_x \otimes H$$

$\therefore$ U is not Clifford

3. Consider the reversible (N)AND



a	b	c	d
0	0	0	0
1	0	0	0
0	1	0	0
1	1	0	1
0	0	1	1
1	0	1	1
0	1	1	1
1	1	1	0

Consider $U^\dagger \mathbb{1}\otimes\mathbb{1}\otimes\sigma_z U$

this gives a phase -1 when $d=1$, so when acting on $|abc\rangle$

$$U^\dagger \mathbb{1}\otimes\mathbb{1}\otimes\sigma_z U = |000\rangle\langle 000| + |100\rangle\langle 100| + |010\rangle\langle 010| + |111\rangle\langle 111| - |110\rangle\langle 110| - |001\rangle\langle 001| - |101\rangle\langle 101| - |011\rangle\langle 011|$$

If this could be decomposed into Paulis, clearly it would be of the form

$$U^\dagger \mathbb{1} \otimes \mathbb{1} \otimes \sigma_z U = \sigma_z^\alpha \otimes \sigma_z^\beta \otimes \sigma_z^\gamma, \quad \alpha, \beta, \gamma \in \{0,1\}$$

but nothing of this form works, so U is not Clifford