Quantum Computation 2017

Sheet 3 Abgabe 04/12/2017

1. Trotter and friends

Consider the following relations for the Hamiltonians (and hence Hermitian matrices) A and B. These are the basic tools you can use to investigate exactly how to simulate arbitary Hamiltonians yourself.

(a) Show that you can simulate the Hamiltonian $A + B$ for the time interval Δt with an $O(\Delta t^2)$ error by proving the following.

$$e^{i(A+B)\Delta t} = e^{iA\Delta t} e^{iB\Delta t} + O(\Delta t^2). \tag{1}$$

(b) Show that repeating this N times raises the error to $O(N\Delta t^2)$, as long as the time interval is small enough in comparison to the number of repetitions,

$$\left[e^{i(A+B)\Delta t} + O(\Delta t^2)\right]^N = e^{i(A+B)N\Delta t} + O(N\Delta t^2). \tag{2}$$

(c) Show that third order Trotter-based approximation offers better accuracy

$$e^{i(A+B)\Delta t} = e^{iA\Delta t/2} e^{iB\Delta t} e^{iA\Delta t/2} + O(\Delta t^3). \tag{3}$$

2. Decomposition of single qubit rotations

(a) For a single qubit, we don't need Trotterization. Show that any single qubit unitary can be expressed in the form

$$U = e^{-i\delta} e^{-i\sigma_z \gamma} e^{-i\sigma_y \beta} e^{-i\sigma_z \alpha}. \tag{4}$$

(b) We can also build single qubit unitaries if we only have rotations around two axes that are not orthogonal. Show that any single qubit unitary can be expressed in the form,

$$U = e^{-i\delta} \prod_j e^{-i\vec{v}\cdot\vec{\sigma}\beta_j}\, e^{-i\vec{u}\cdot\vec{\sigma}\alpha_j}, \tag{5}$$

where $\vec{u}$ and $\vec{v}$ are non-parallel (not necessarily orthogonal) unit vectors. The product over j simply means that you can repeatededly rotate around the two axes an arbitrary number of times, and the angles of rotation for each can be different.