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a) clearly $e^{iHt} = 1 + iHt + O(t^2)$

and $1 + iHt = e^{iHt} + O(t^2)$

So for a product

$$e^{iAt} e^{iBt} = 1 + iAt + iBt + O(t^2)$$

$$= e^{i(A+B)t} + O(t^2)$$

c) similar stuff

b) Using the binomial theorem

$$(x+y)^n = x^n + \sum_{k=1}^{n} \binom{n}{k} x^{n-k} y^k$$

we find, for hermitian C

$$\left[e^{iC\Delta t} + O(\Delta t^2)\right]^N = e^{iCN\Delta t} + \sum_{k=1}^{N} \binom{N}{k} e^{iC\Delta t(N-k)} \, O\left(\frac{1}{\Delta t^{2k}}\right)$$

Note that $e^{iC\Delta t(N-k)} = O(1)$, and

$$\sum_{k=1}^{N} \binom{N}{k} e^{iC\Delta t(N-k)} \, O\left(\frac{1}{\Delta t^{2k}}\right) = \sum_{k=1}^{N} O\left(N^k\right) O(1) \, O\left(\frac{1}{\Delta t^{2k}}\right) = \sum_{k=1}^{N} O\left(\left[\frac{N}{\Delta t^2}\right]^k\right)$$

For $N < \Delta t^2$, this is dominated by lowest order

$$\sum_{k=1}^{N} O\left(\left[\frac{N}{\Delta t^2}\right]^k\right) = \frac{N}{\Delta t^2}$$

So finally

$$\left[e^{iC\Delta t} + O(\Delta t^2)\right]^N = e^{iCN\Delta t} + O\left(\frac{N}{\Delta t^2}\right)$$

When $N < \Delta t^2$

2

a) Euler angles. Standard stuff

b) I can explain better in person