Quantum Information 2018
Sheet 11

1. $2D$ Ising model

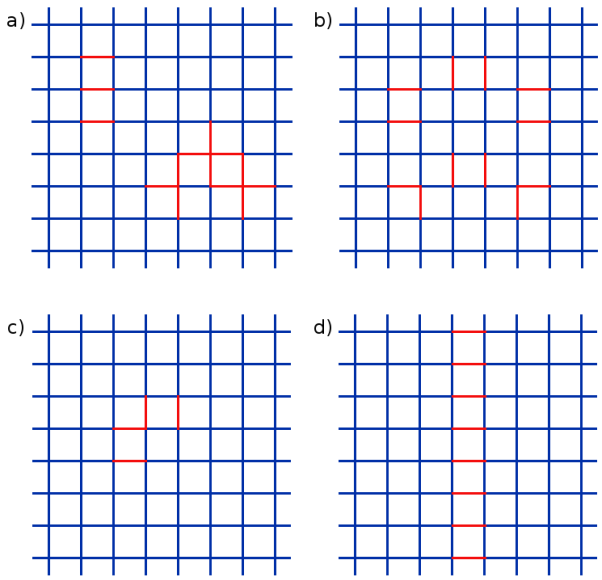
The figure shows four examples of a $2D$ Ising model defined on a square lattice. The boundary conditions are periodic, and a qubit resides on each vertex. The Hamiltonian for the model is

$$H = -\sum_{\langle i,j \rangle} J_{i,j}\ \sigma_z^i \sigma_z^j. \tag{1}$$

The coupling $J_{i,j}$ is $+J$ for blue links and $-J$ for red ones, where $J > 0$.

For each of the four models, find: (i) the locations of frustrated plaquettes; (ii) all ground states; (iii) the ground state energy. Also, show that case (d) is frustrated, despite having no frustrated plaquettes.

Note that cases (a) and (b) each have two degenerate ground states, whereas (c) and (d) have more.



as $N \to \infty$ for $T = 0$ and $T > 0$.

2. Energy Barriers for Ising models

For a completely ferromagnet Ising model there are two ground states (all $|0\rangle$'s and all $|1\rangle$'s). Starting at one ground state, consider flipping spins one at a time until you reach the other ground state. Depending on the order in which you flip the spins, the energy of the intermediate states will be different.

a) For the 2D Ising model on a lattice with L^2 spins, what order of spin flips will keep the energy at the minimum possible at all times? What is the maximum energy throughout this process?

b) Do the same for the 1D Ising model, defined in the same way but on a line of L spins.