Quantum Information 2018
Sheet 6

1. Applying Peres-Horodecki

Consider the entangled state,

$$|\,\phi\rangle_{AB} = \alpha\,|\,00\rangle + \beta\,|\,11\rangle\,,$$

where α and β are real and non-negative.

a) Show that the partial transpose of the density matrix has at least one negative eigenvalue for all $\alpha, \beta \neq 0$.

b) A σ_x is applied to qubit A with probability p. Write down the corresponding density matrix, and determine the value of $p(\alpha, \beta)$ at which the state is no longer entangled.

2. Concurrence

For states of the form $|\,\phi\rangle_{AB}$:

a) Find the entanglement entropy, E.

b) Find the concurrence, C.

c) Show that these satisfy the relation,

$$E = H\left(\frac{1+\sqrt{1-C^2}}{2}\right),$$

where $H(\cdot)$ is the Shannon entropy.

3. Separable states from classical correlations

The state of two systems A and B is said to be separable if it can be written in the form

$$\rho_{AB} = \sum_{j,k} P(j,k)\rho_A^j \otimes \rho_B^k. \qquad (1)$$

In lecture it was claimed that all such states can be generated from an initially uncorrelated state $|\,0\rangle_A \otimes |\,0\rangle_B$ by LOCC. However, they can also be generated from this state using local operations and preshared classical correlations (no communication required). Show that this is true, and find the classically correlated variables that could generate the state of Eq (1).