ZX - Calculus

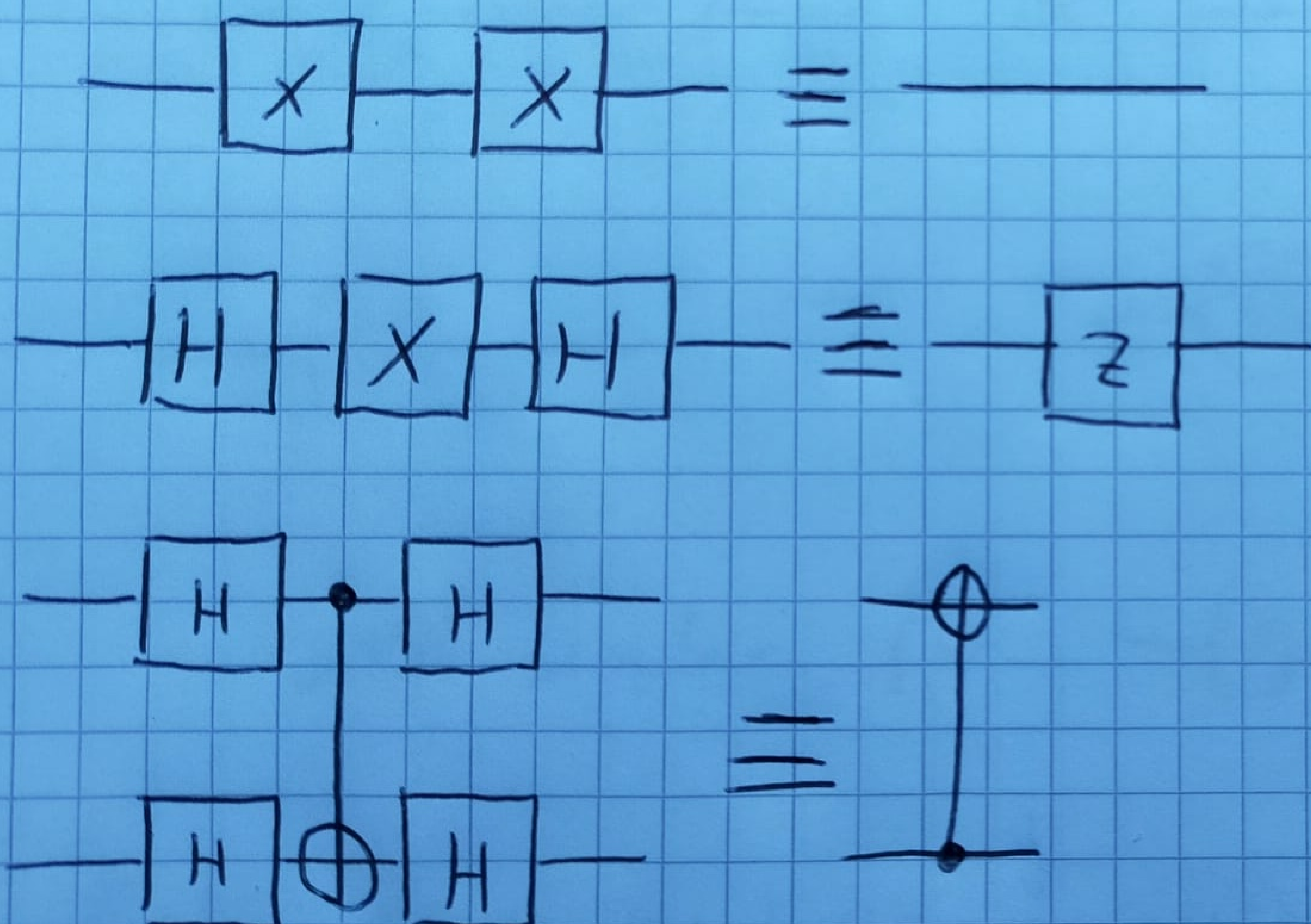
We often need to 'transpile':

- turn a circuit into an equivalent circuit

Reasons:

- Get a shorter circuit
- Adapt to hardware constraints
- Incorporate redundancy required for fault-tolerance

To transpile we make use of circuit identities



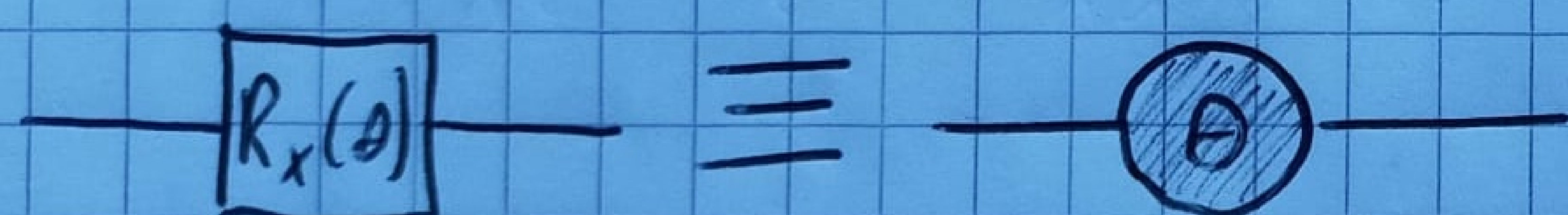
etc

But this can be hard!

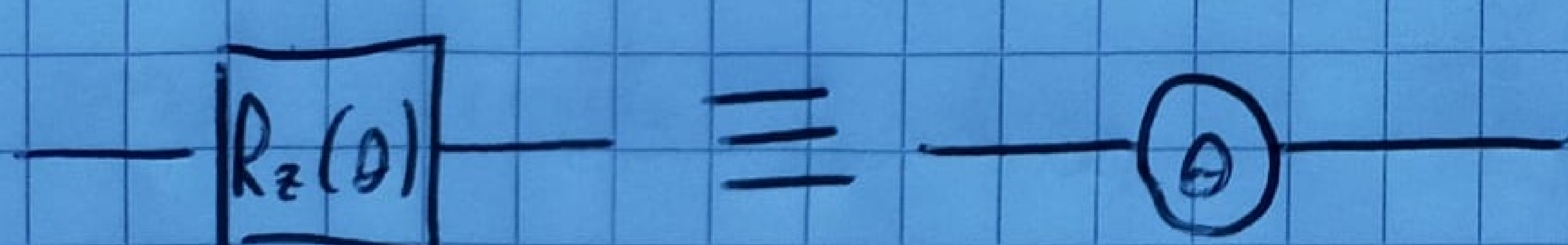
Could an alterative to circuits make it easier?

- An equivalent language that is easier to manipulate

First, we replace R_x and R_z with blobs

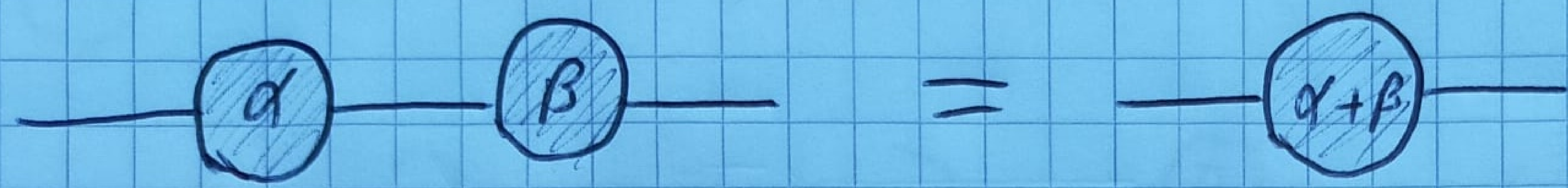


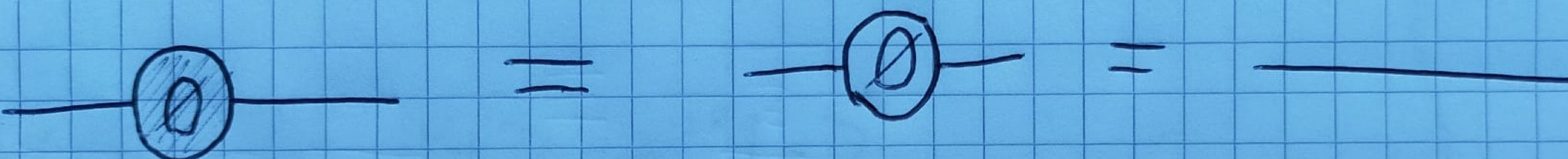
$$= |+\rangle\langle+| + e^{i\theta}|-\rangle\langle-|$$

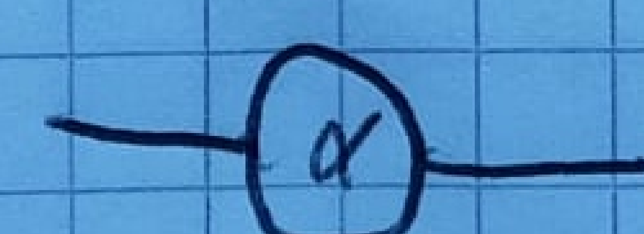


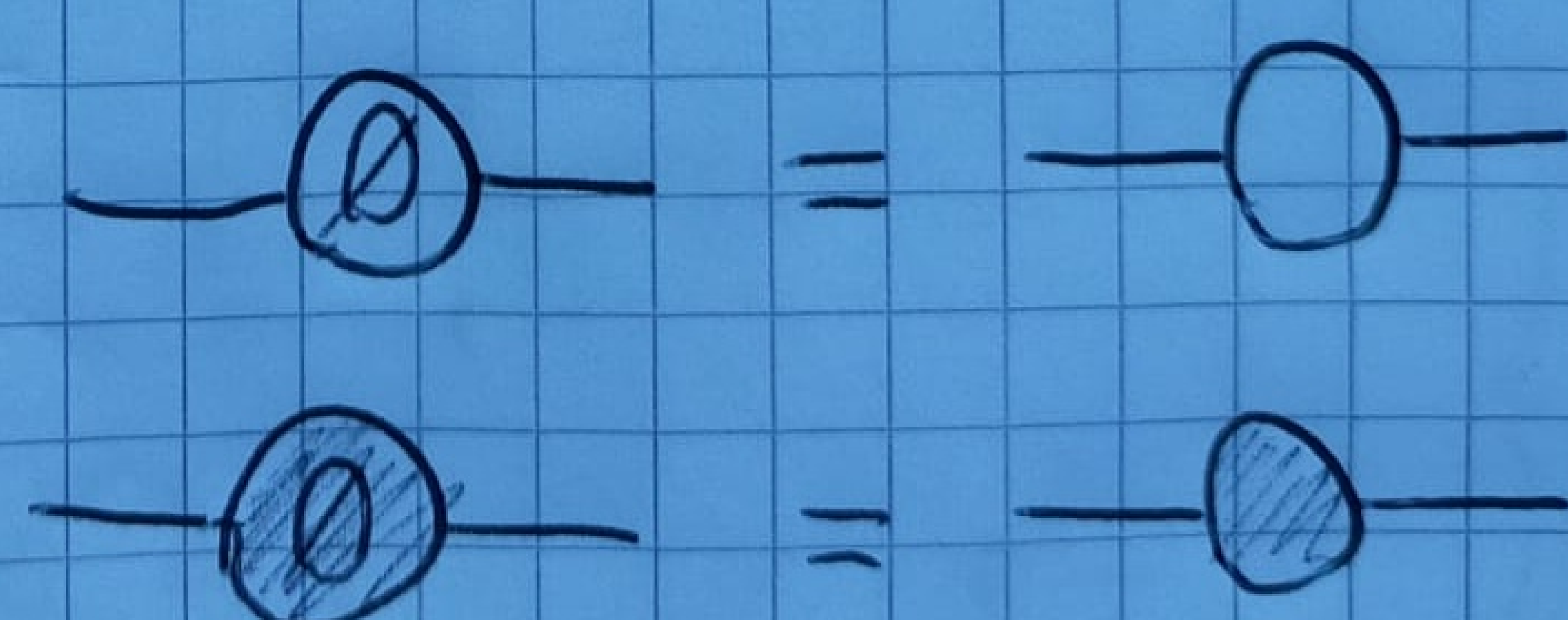
$$= |0\rangle\langle 0| + e^{i\theta}|1\rangle\langle 1|$$

Already we can write down a few circuit identities with these blobs



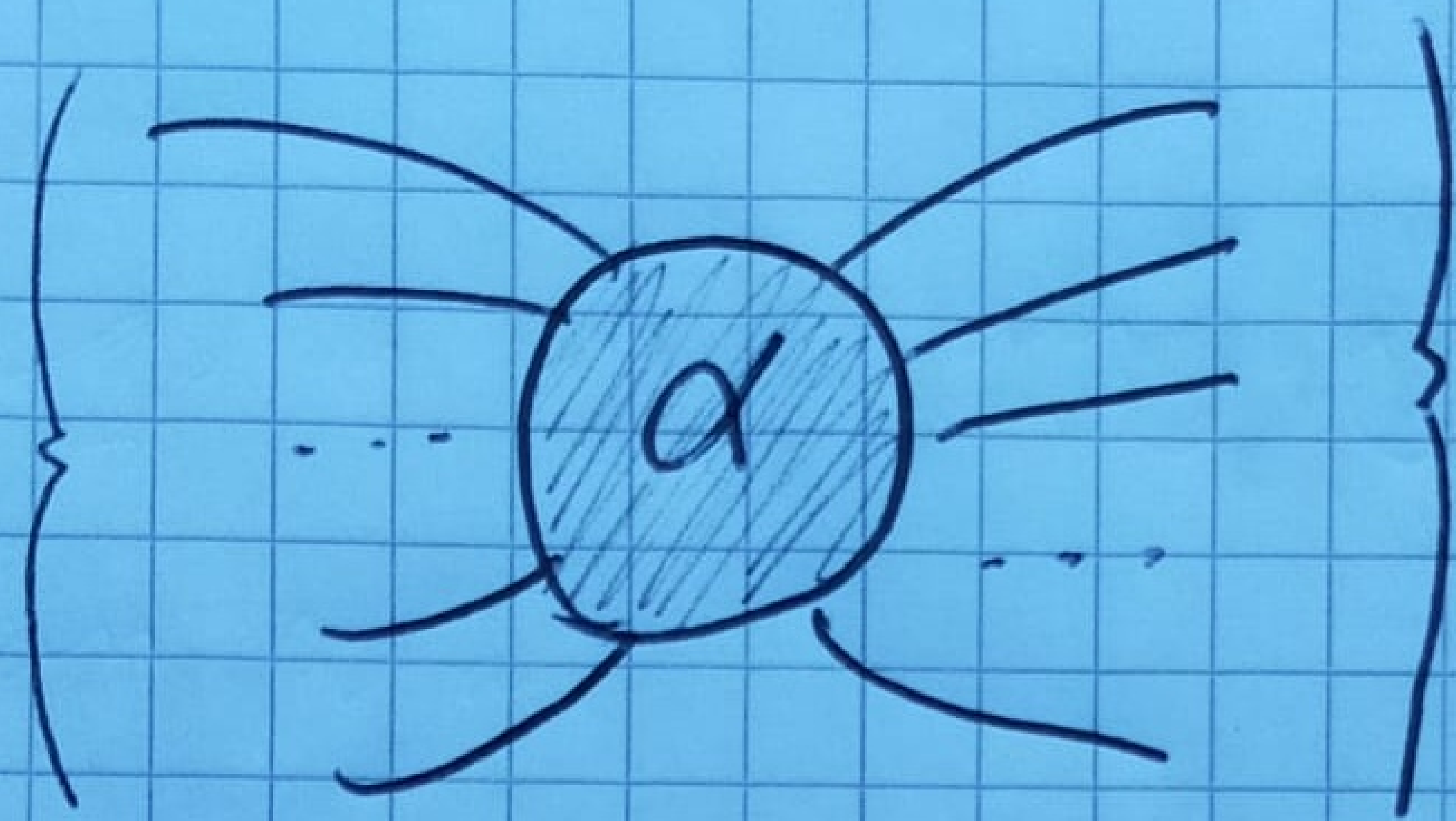


and similar for 

Note: From now on 

Now let's generalize to 'spiders'

m inputs



n outputs

$$= |+\rangle^{\otimes n}\langle +|^{\otimes m} + e^{i\alpha}\, |-\rangle^{\otimes n}\langle -|^{\otimes m}$$

and similar for 

One-legged examples:

 $= |0\rangle + e^{i\alpha}|1\rangle$

$\therefore$  $= |+\rangle$,  $= |-\rangle$

 $= |+\rangle + e^{i\alpha}|-\rangle$

$\therefore$  $= |0\rangle$,  $= |1\rangle$

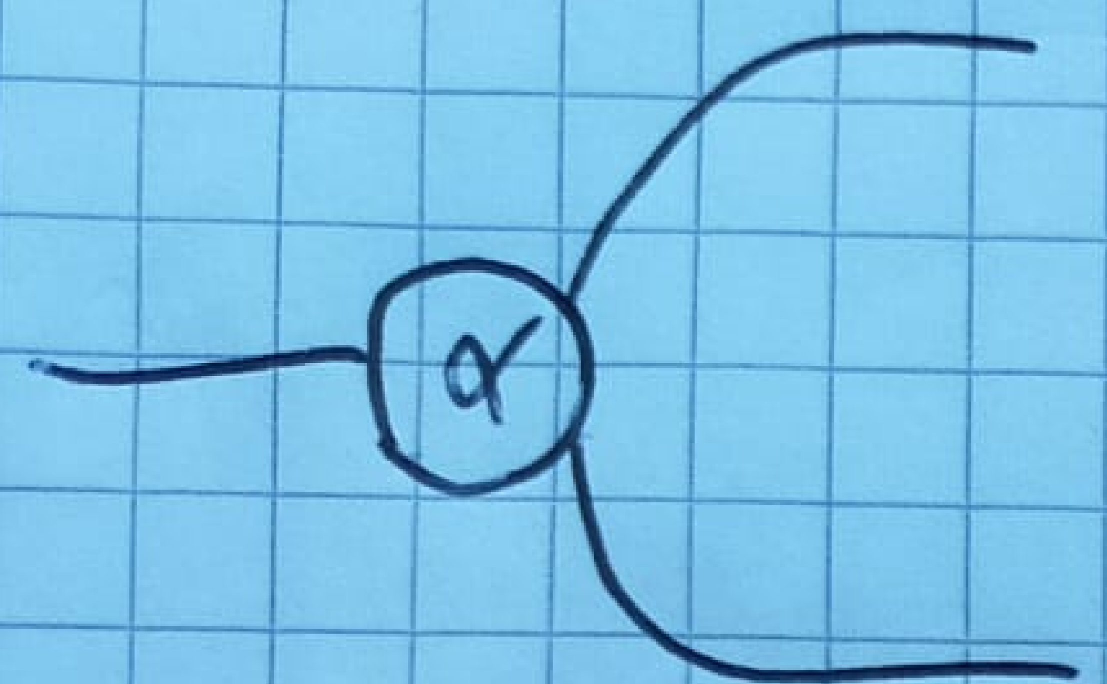
Preparation

Similarly  $\equiv$ 

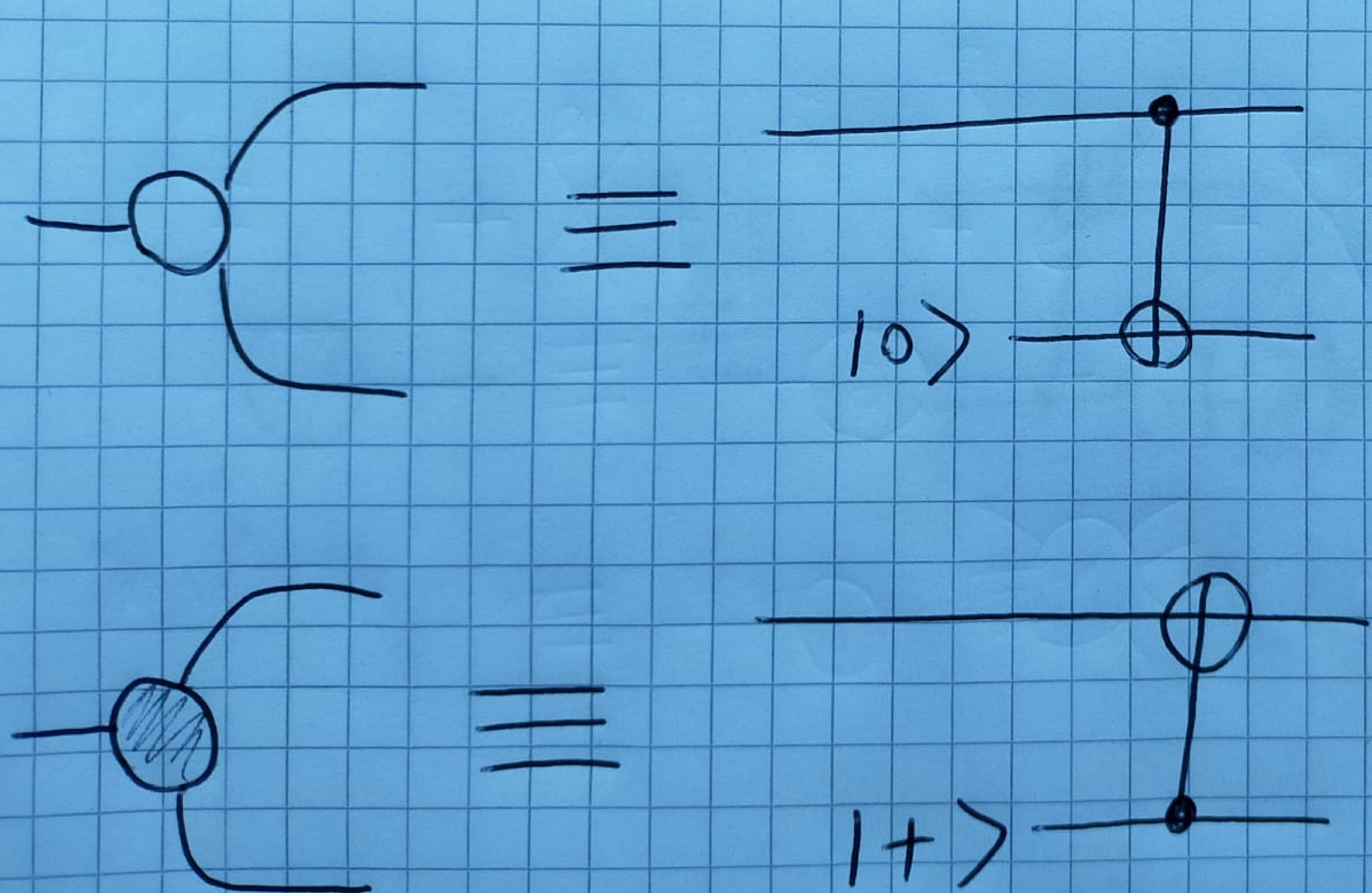
$= \langle b|$ etc

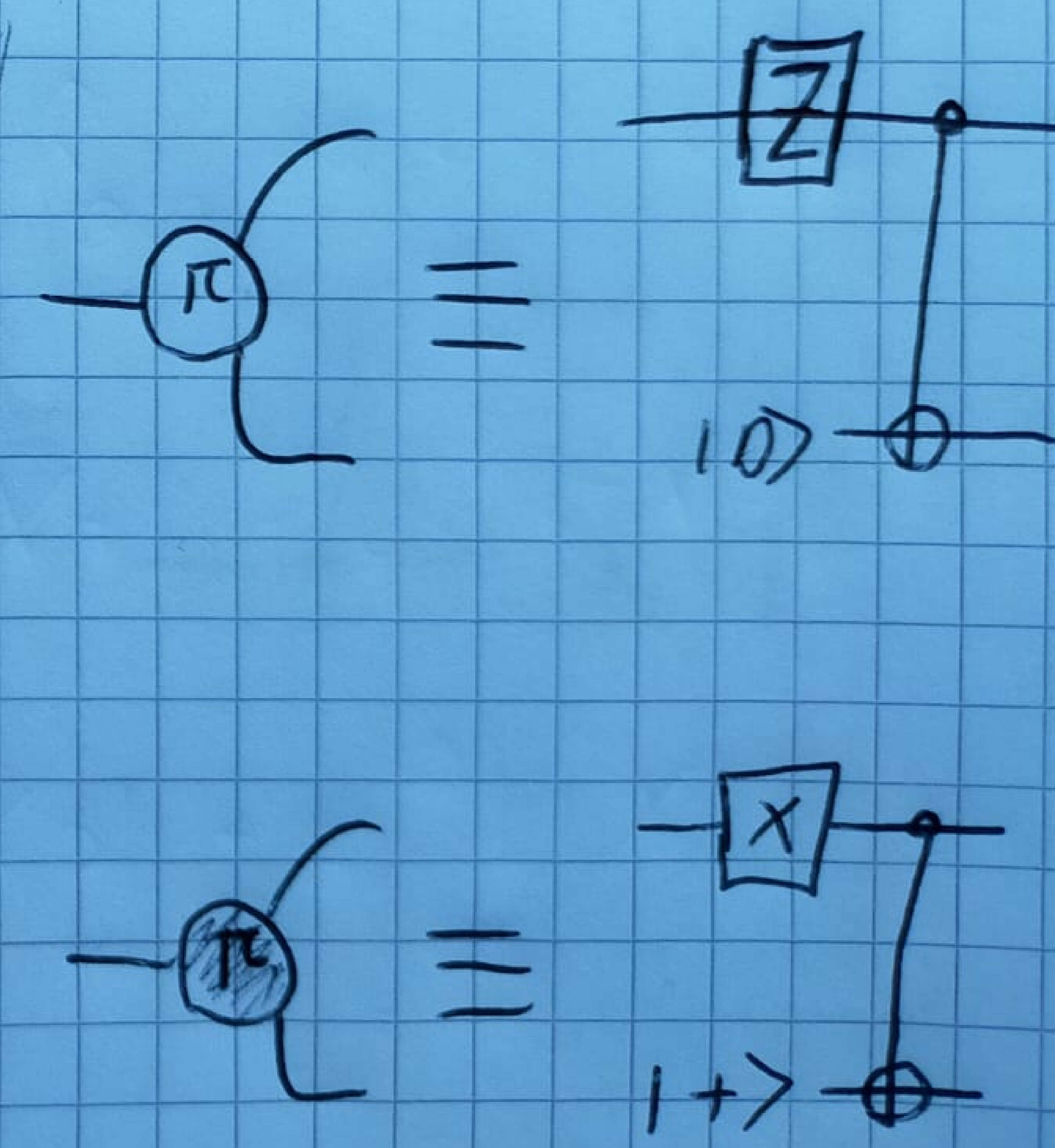
Measurement

Three legged examples

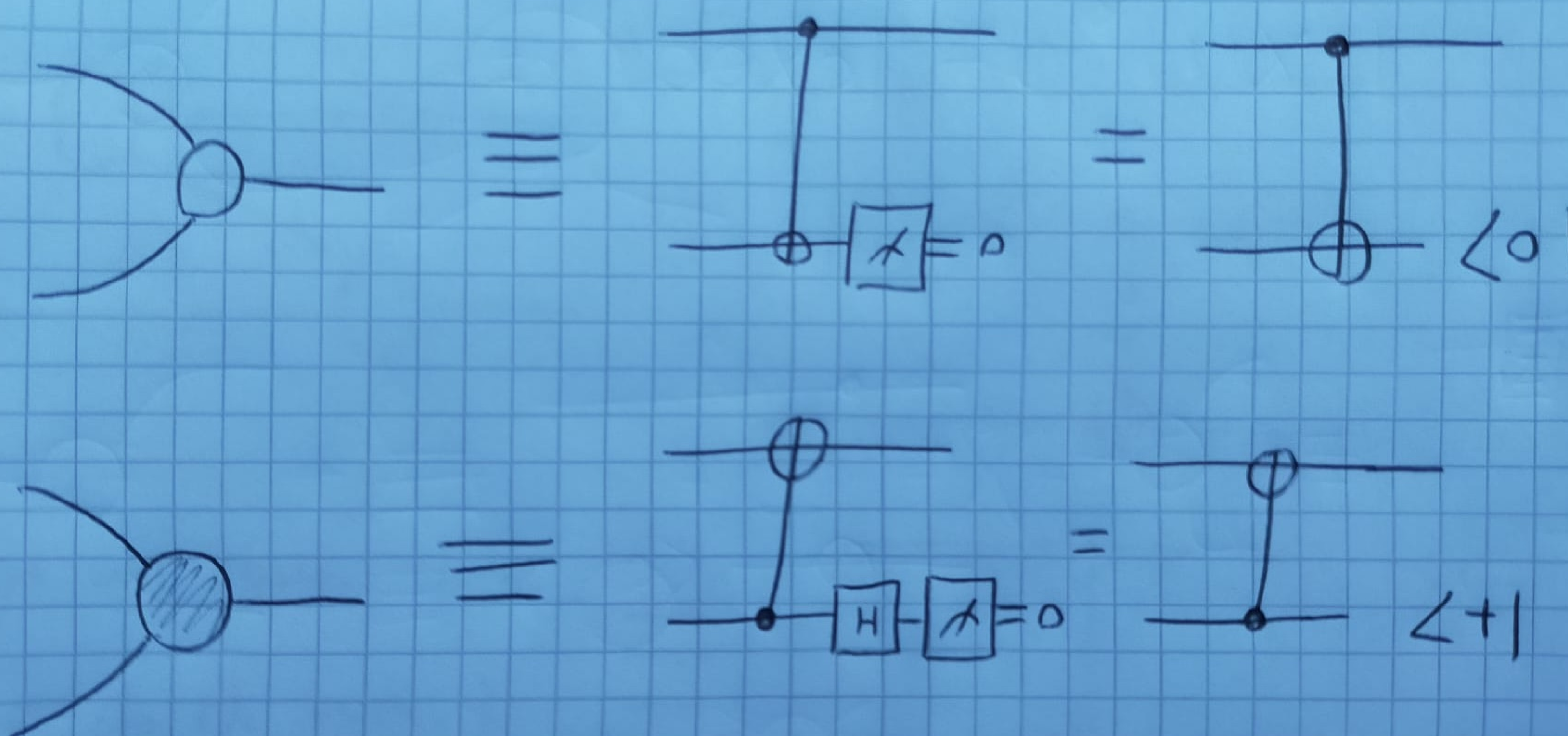


$$= |00\rangle\langle 0| + e^{i\alpha} |11\rangle\langle 1|$$

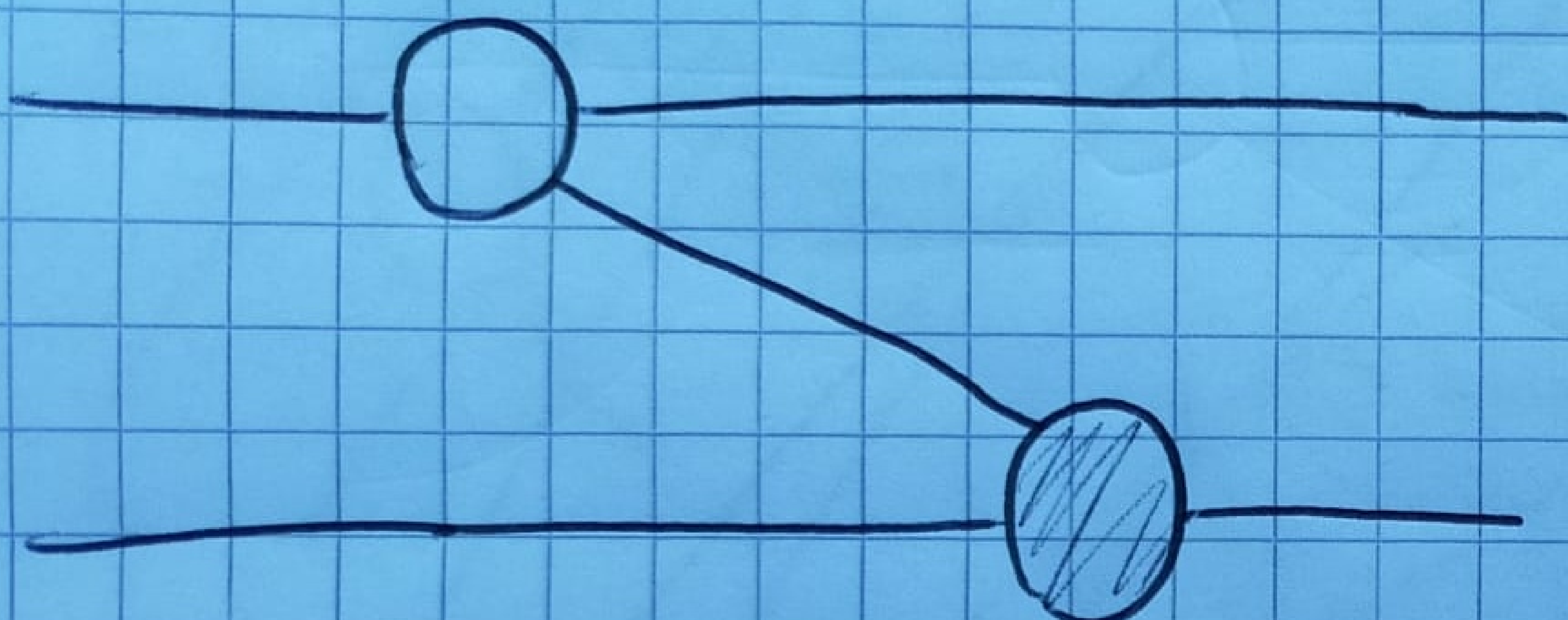




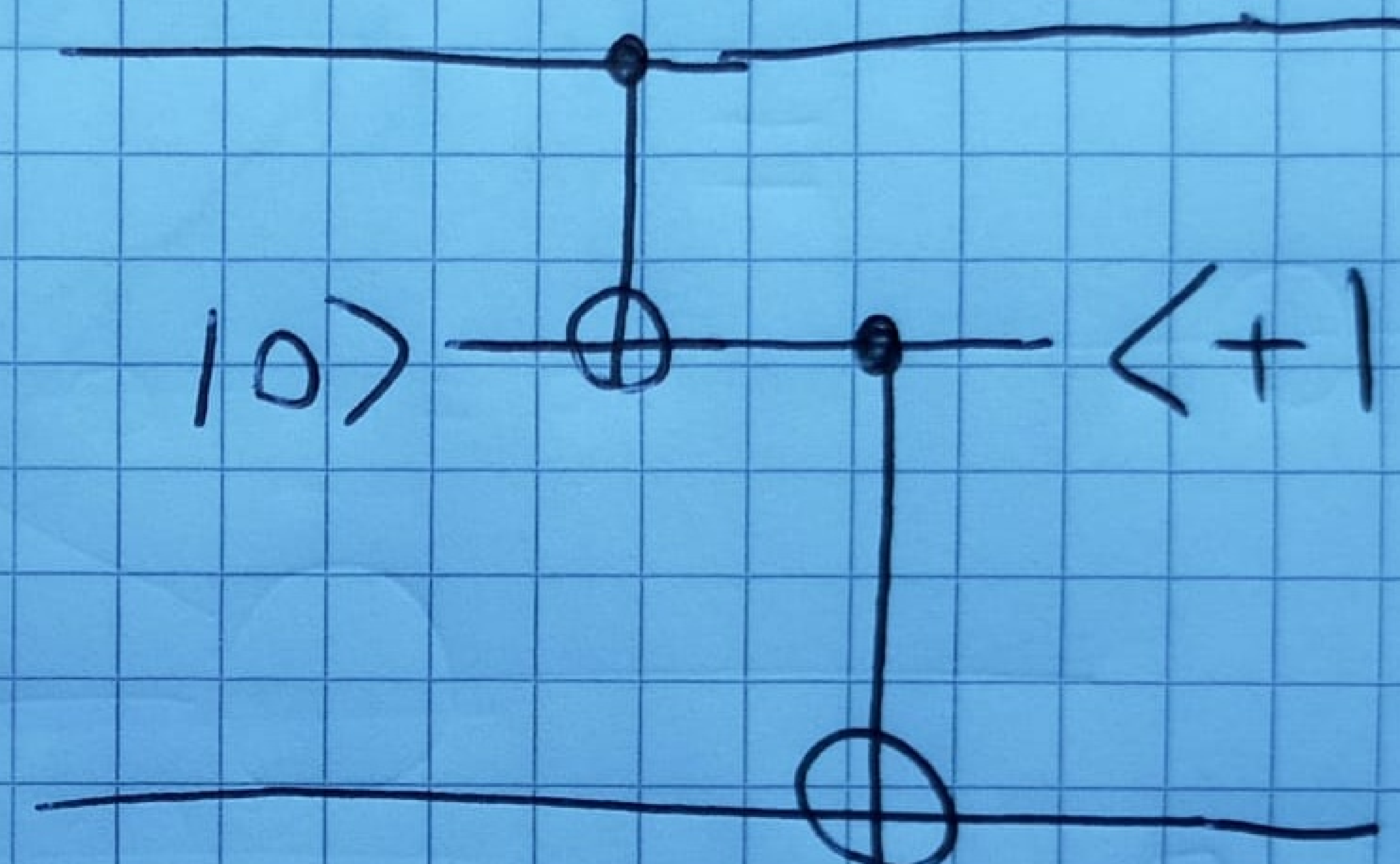
We'll consider the other direction as post selection



Now let's chain two together and see what happens

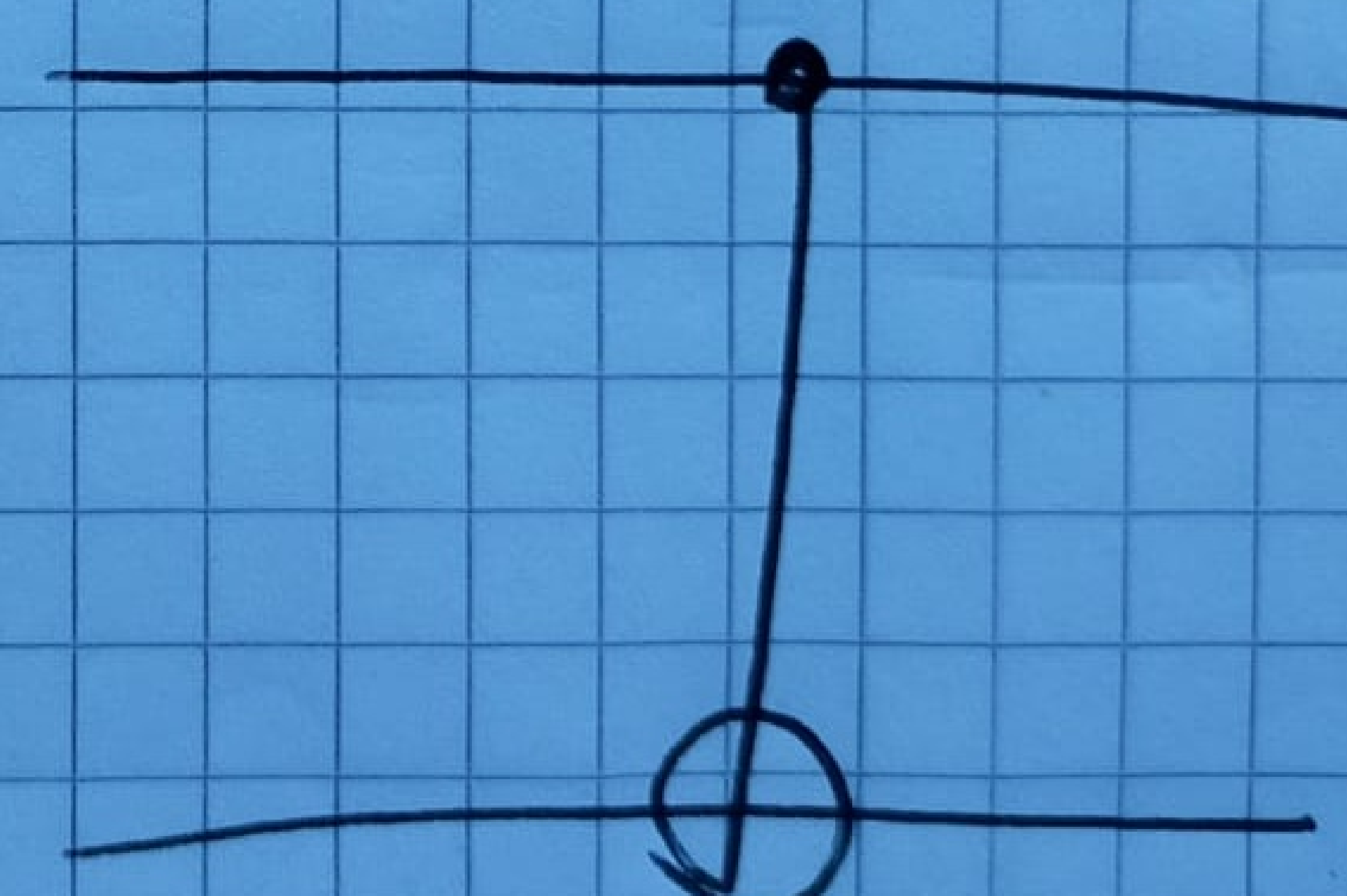


$\equiv$



(Not an obvious Jump, but do the maths)

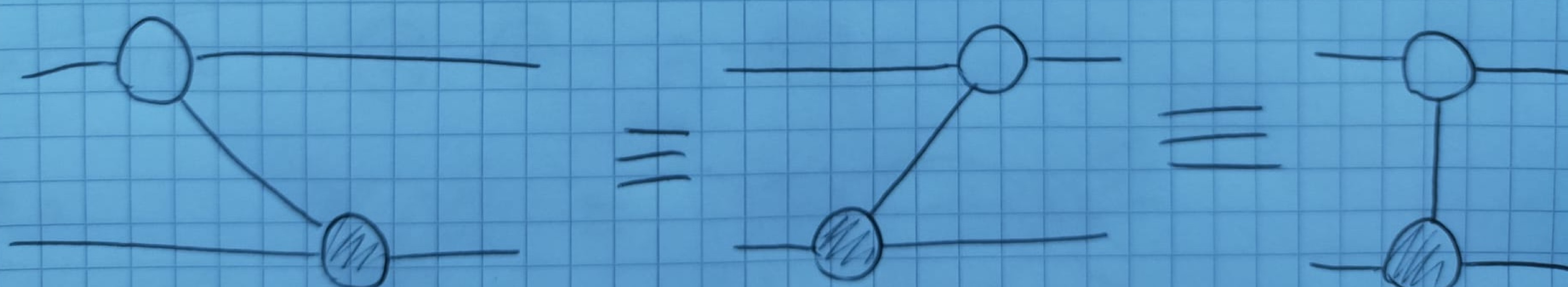
$\equiv$



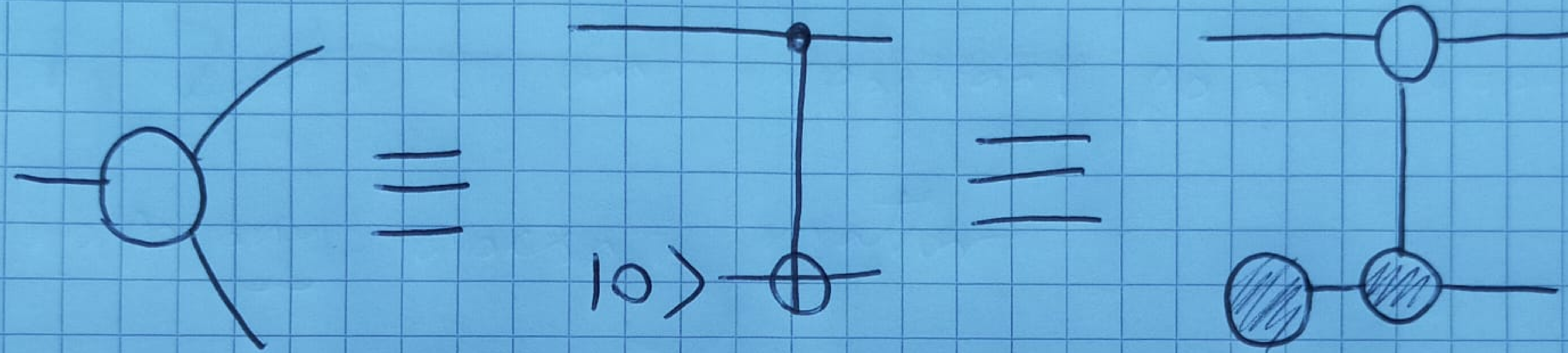
Also 

Which leads to an important point about ZX:

Topology doesn't matter

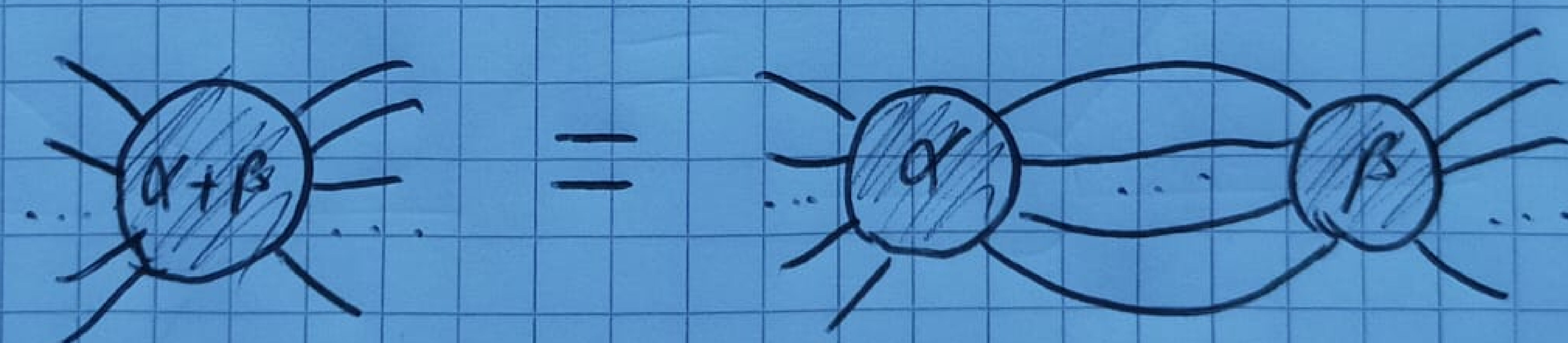


With this in mind



Generalize

to



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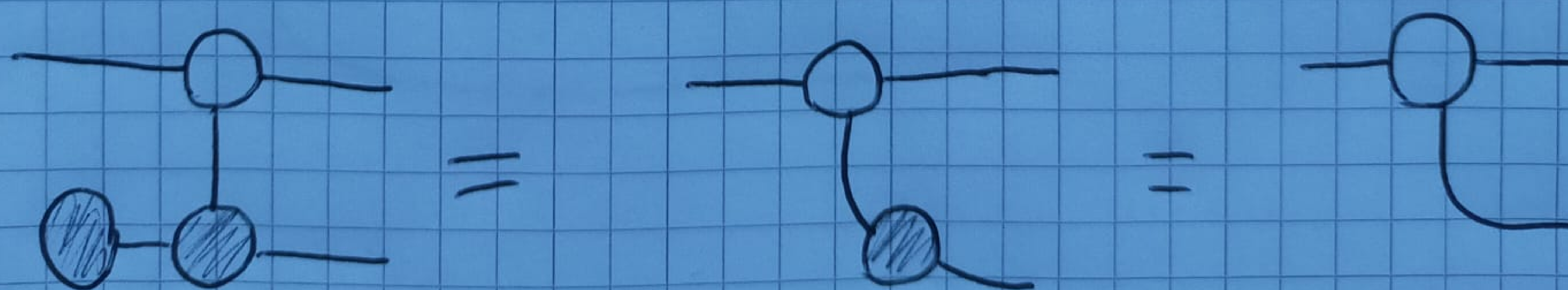
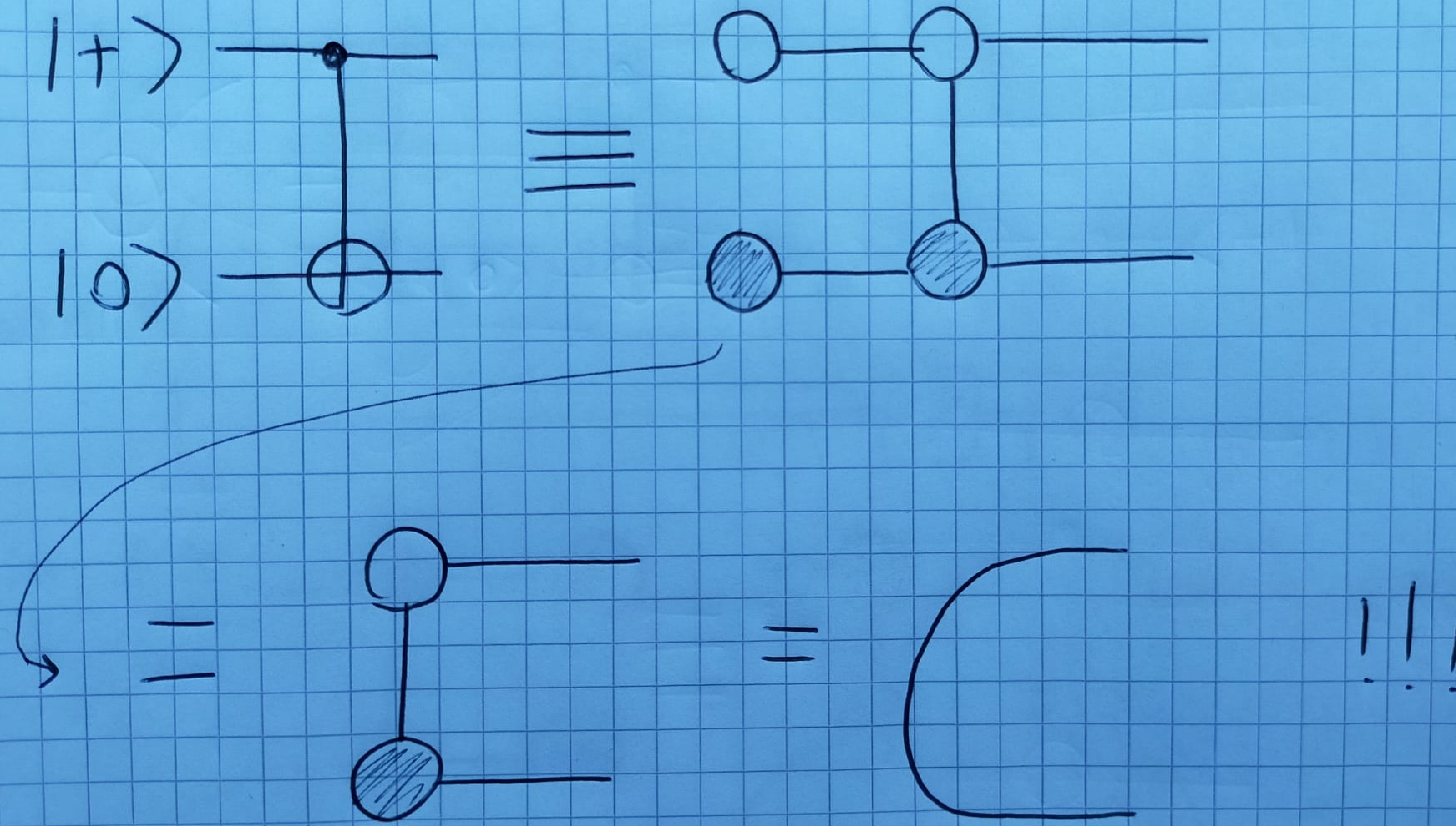
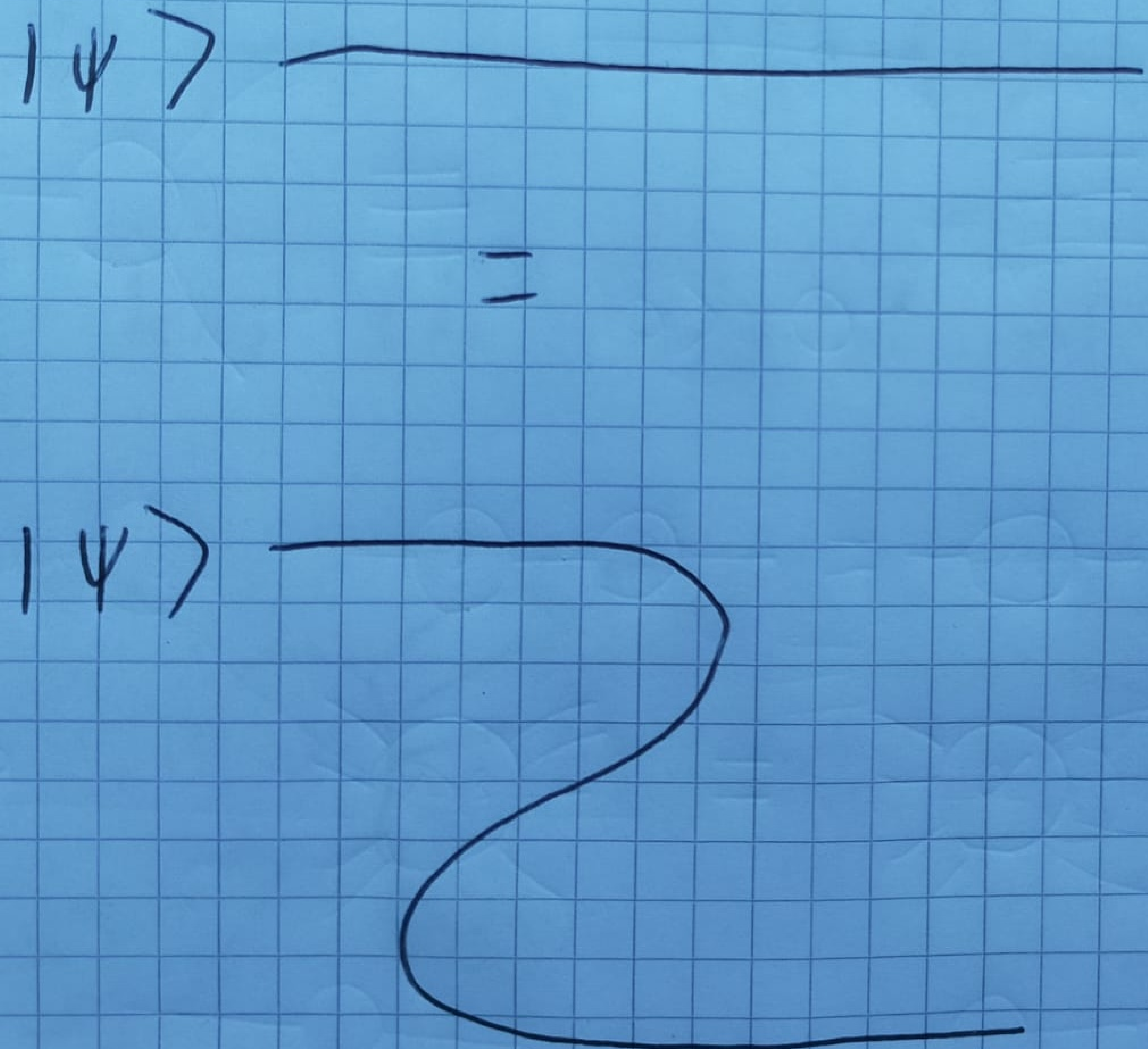


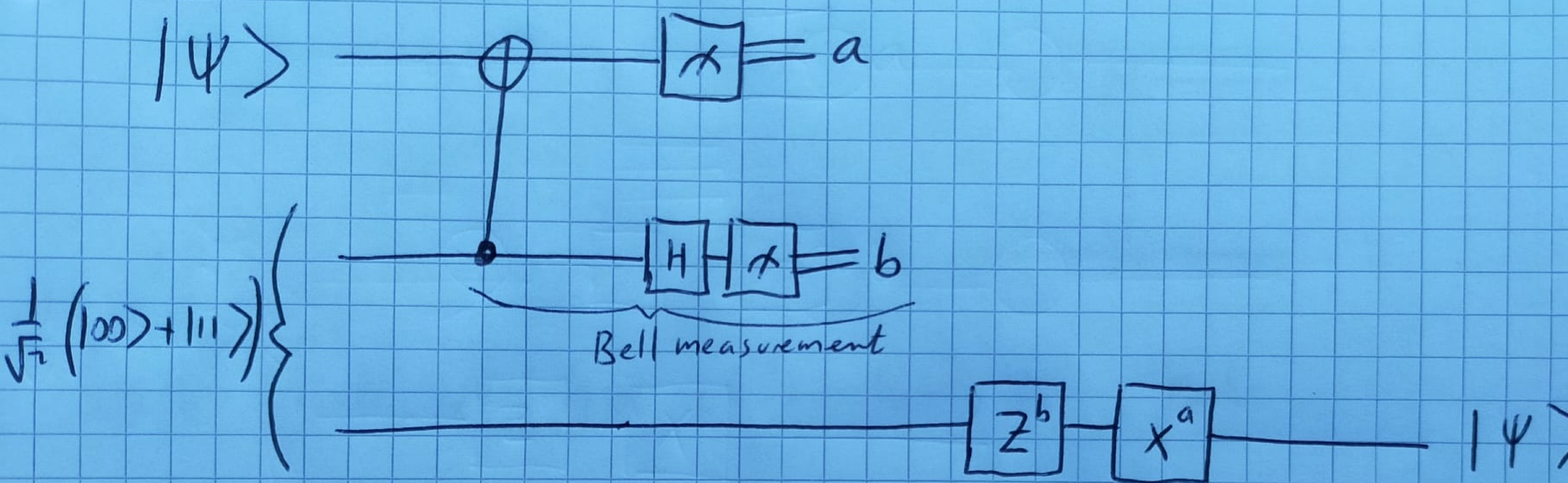
Now let's consider Bell pairs



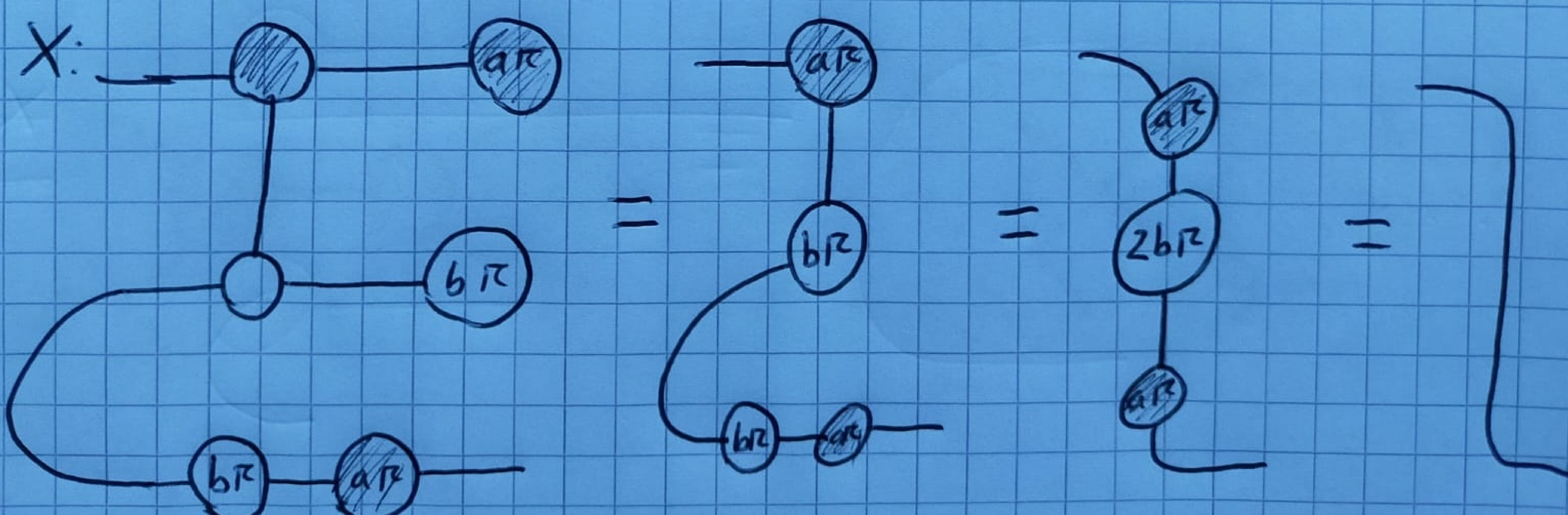
This leads us to a famous quantum protocol



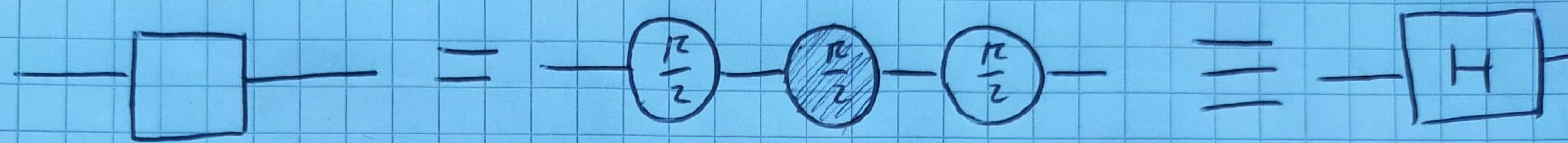
In more detail, the teleportation circuit is



In ZX:

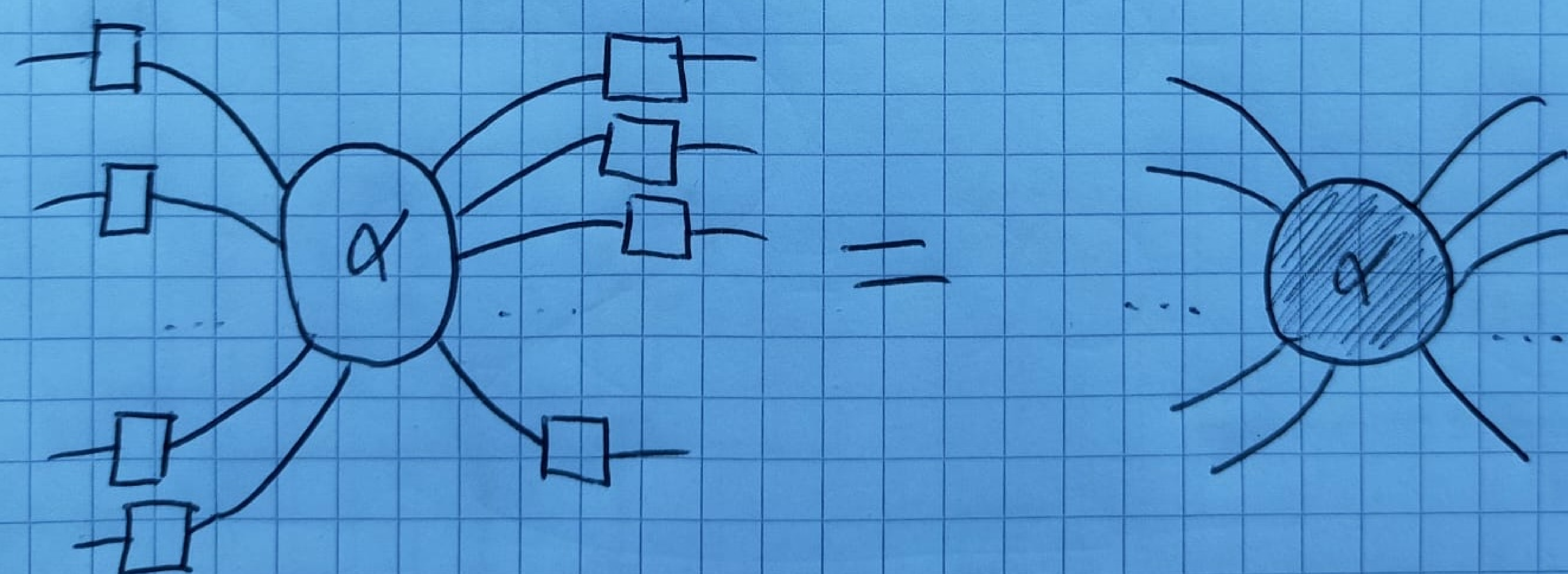


Another useful element is the Hadamard

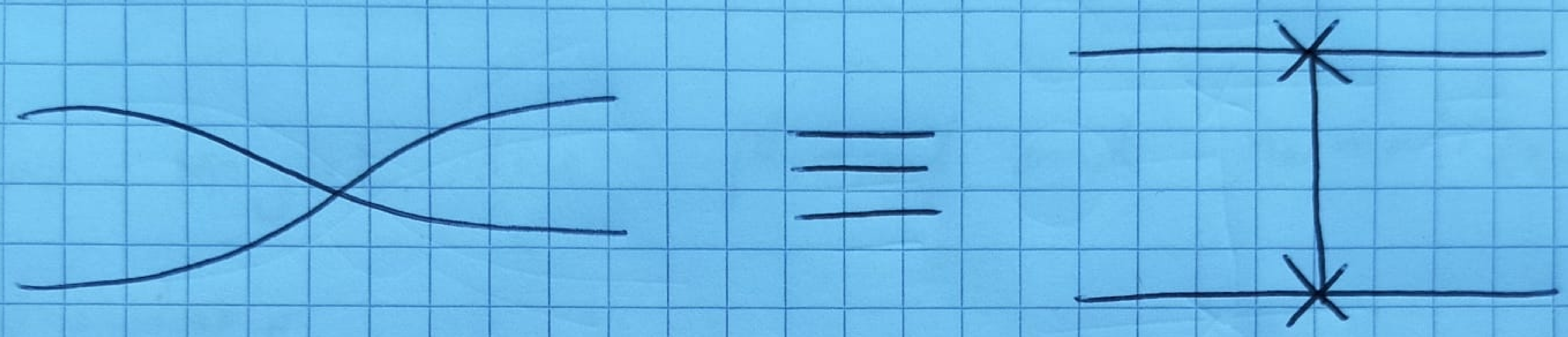


This has the effect 

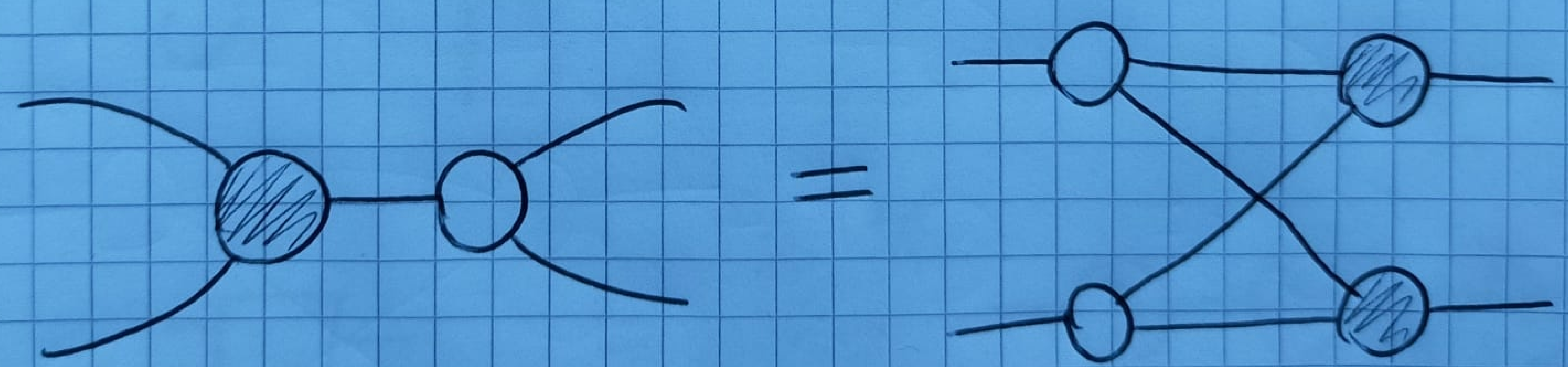
and



Also, the SWAP gate



With which we get the bialgebra rule



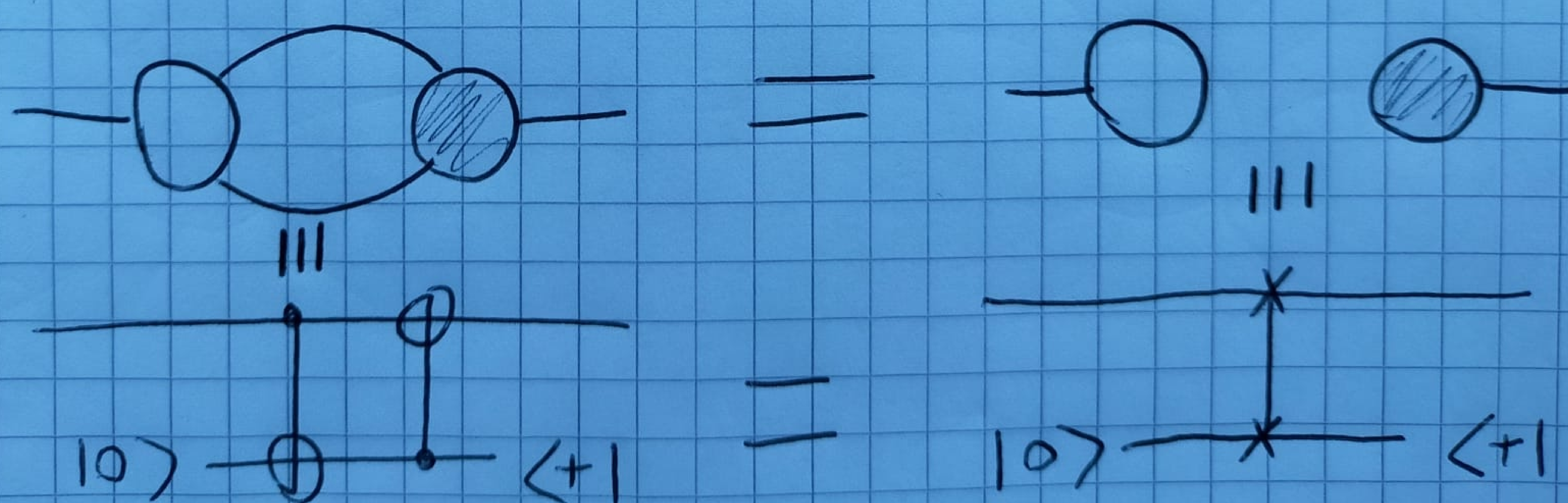
(no nice story)

Some final rules

Copy rule:

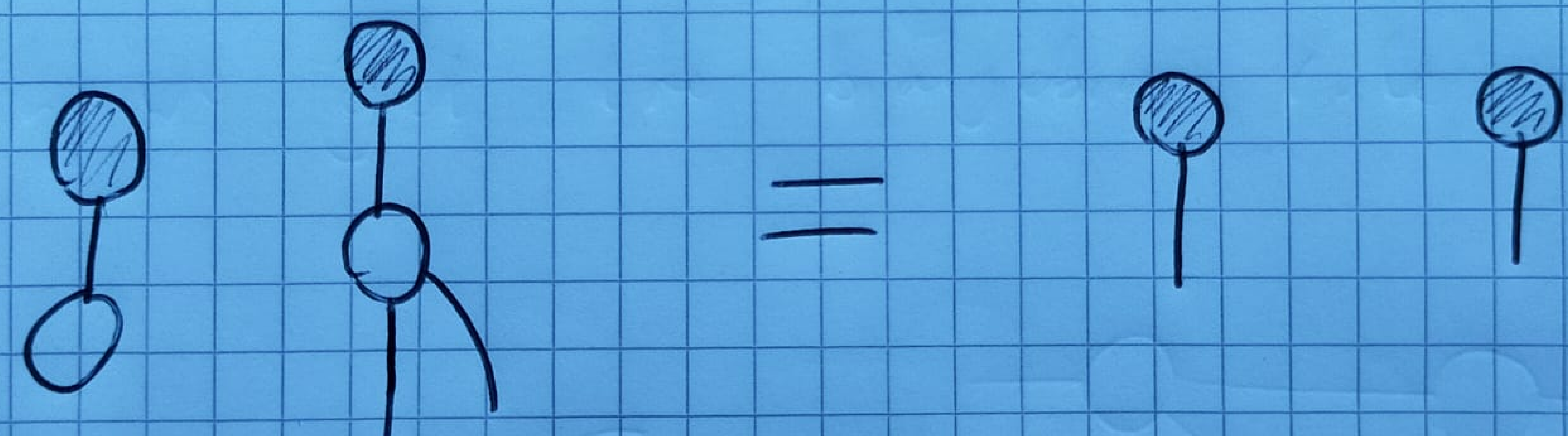
(follows from the copying op)

Hopf rule:



Note that we've gone phaseless and unnormalized here

You might see phased and normalized versions elsewhere



$$= \frac{e^{i\pi/4}}{\sqrt{2}}$$