Quantum Computation 2017

Sheet 2 Abgabe 27/11/2017

1. Building systems out of qubits

Consider a system composed of n_1 spin-1 particles for which we define the basis states $|-1\rangle$, $|0\rangle$ and $|1\rangle$, and $n_{3/2}$ spin-3/2 particles with basis states $|-3/2\rangle$, $|-1/2\rangle$, $|1/2\rangle$ and $|3/2\rangle$.

(a) What is the minimum number of qubits, n, required simulate such a system?

(b) What is n if the spins are not allowed to share qubits, such that each spin is simulated by its own set of qubits?

(c) Using big-O notation, compare the scaling of n with n_1 and $n_{3/2}$ for both cases. Which scales more efficiently, or are they the same?

(d) For $n_1 = 1$ and $n_{3/2} = 0$ we would require two qubits. The basis states of the single spin-3/2 could then be identified with qubit states according to $|00\rangle = |-1\rangle$, $|01\rangle = |0\rangle$, $|10\rangle = |+1\rangle$. The qubit state $|11\rangle$ is then not involved in the simulation.

Find a way to identify the basis states of four qubits with the basis states for $n_1 = n_{3/2} = 1$.

2. Clifford gates

(a) Determine the effects of the following single qubit unitary when applied to the X, Y and Z basis states.

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

(b) Determine the Bloch sphere rotation that this corresponds to.

(c) Show that this is a Clifford gate, and determine how it manipulates the expectation values of single qubit Paulis.

(d) Find the Bloch sphere rotations corresponding to the F, G and H gates from the tutorial.