Quantum Information 2018
Sheet 2

1. Single Qubit Tomography

Prepare four different single qubit states using IBMs Quantum Experience: $|0\rangle$, $|1\rangle$, $|+\rangle$ and $|-\rangle$. Do this on the real device if possible (which will result in noisy results).

For all states, measure in the X, Y and Z bases and use the results to calculate the expectation values $\langle X\rangle$, $\langle Y\rangle$ and $\langle Z\rangle$. With these, construct the corresponding density matrix for each of the four states using the relation,

$$\rho = \frac{1 + \langle X\rangle X + \langle Y\rangle Y + \langle Z\rangle Z}{2}. \tag{1}$$

(a) Check whether the density operators are indeed $\mathrm{Tr}(\rho_i) = 1$ and Hermitian and positive, as required.

(b) Diagonalize the density operators. Comment on their similarities and differences with the intended states$|0\rangle$, $|1\rangle$, $|+\rangle$ and $|-\rangle$.

2. Relaxation and Dephasing

The Quantum Experience shows T_1 and T_2 times for all qubits on its devices. The *relaxation time* T_1 describes the timescale of the decay of the excited state $|1\rangle$ into the groundstate $|0\rangle$. Whether $|1\rangle$ and $|0\rangle$ actually correspond to excited and ground state depends of course on the implementation.

To see the effect of relaxation, prepare the $|1\rangle$ state and measure it in the Z basis after a number of identity gates. Identity gates simply force the system to wait with the measurement. Experiment with different wait times, e.g. 1,20,30 and 40 and interpret the result. If the data looks like an exponential, try to fit it to $\exp(-\frac{n\delta_t}{T_1})$, where n is the number of identity gates you applied, and reconstruct the wait time per identity δ_t.

The *dephasing time* T_2 describes the timescale of the decay of superpositions into mixtures, e.g. $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ to $|0\rangle\langle 0| + |1\rangle\langle 1|$. Demonstrate the effect by starting with $|0\rangle$, applying H to get $|+\rangle$ and then applying S and $S^\dagger$ with n identity gates in between. Finally, rotate the state back with H and measure the result. What would you expect? What do you actually see happening as a function of n? If the data looks like an exponential, try to fit it to $\exp(-\frac{n\delta_t}{T_2})$ as above.

Use the same (physical) qubit for each calculation to keep the T_1 and T_2 times the same.