Quantum Information 2018
Sheet 4

Shor Code

The Shor (or 9 Qubit) code is a method for quantum error correction based on the classical repetition code. First a logical qubit is stored in three physical qubits using the encoding

$$|+\rangle_3 = |+++\rangle\,, \ \ |-\rangle_3 = |---\rangle\,.$$

This protects against errors that try to flip $|+\rangle_3$ to $|-\rangle_3$, and vice-versa. But the corresponding errors for the Z basis states

$$|0\rangle_3 = \frac{1}{\sqrt{2}}(|+++\rangle + |---\rangle), \ \ |1\rangle_3 = \frac{1}{\sqrt{2}}(|+++\rangle - |---\rangle)$$

become more likely. To deal with this we take three of these logical qubits and use them (like the original physical qubits) to encode a single logical qubit. This uses the encoding:

$$|0\rangle_9 = |0\rangle_3 \otimes |0\rangle_3 \otimes |0\rangle_3\,, \ \ |1\rangle_9 = |1\rangle_3 \otimes |1\rangle_3 \otimes |1\rangle_3\,.$$

The end result is then a code that stores one logical qubit in 9 physical qubits, with stabilizer states

$$|0\rangle_9 = \left[\frac{1}{\sqrt{2}}\left(|+++\rangle + |---\rangle\right)\right]^{\otimes 3}, \ \ |1\rangle_9 = \left[\frac{1}{\sqrt{2}}\left(|+++\rangle - |---\rangle\right)\right]^{\otimes 3}$$

a) Find operators that act as X and Z on the logical qubit. What are the minimal number of qubits these act on?

b) Suppose σ_x errors occur independently on each qubit with probability p_x. What is the probability P_x that a logical X occurs after syndrome measurement and error correction? For simplicity you can determine this only up to lowest order in p_x.

c) Similarly, what is the probability P_z of Z errors, given that σ_z errors occur with probability p_z? For simplicity you can determine this only up to lowest order in p_z.