Quantum Information 2018
Sheet 7

In lecture 6 we looked at Hardy's paradox. In this exercise you will implement this experiment using IBM's Quantum Experience.

For Hardy's paradox we need a state such as

$$|\,\psi\rangle = \frac{1}{\sqrt{3}}\,|\,00\rangle + \frac{1}{\sqrt{3}}\,|\,01\rangle + \frac{1}{\sqrt{3}}\,|\,10\rangle\,.$$

To construct this state, we can first express it using the Schmidt decomposition

$$|\,\psi\rangle = \sqrt{p_0}\,|\,\alpha_0\rangle \otimes |\,\alpha_0\rangle + \sqrt{p_1}\,|\,\alpha_1\rangle \otimes |\,\alpha_1\rangle\,.$$

It is therefore clearly equivalent up to local unitaries to the state

$$\left|\,\tilde{\psi}\right\rangle = \sqrt{p_0}\,|\,00\rangle + \sqrt{p_1}\,|\,11\rangle\,.$$

This can be prepared from the initial $|\,00\rangle$ state using a rotation around the X axis (using $u3(\theta, -\pi/2, \pi/2)$) to create a superposition with the required amplitudes. This can then be turned into an entangled state by using a CNOT to spread the superposition to the other qubit.

This process creates the required entanglement. To get to the state $|\,\psi\rangle$, the single qubit unitary must be applied with the effect

$$U : |\,j\rangle \rightarrow |\,\alpha_0\rangle \quad j \in \{0,1\}$$

Once you have the state $|\,\psi\rangle$, you can start considering the final measurements. We need two measurement bases. One is simply Z measurement, which the QX gives us for free. The other is in the X basis.

Since the QX only does Z measurements. To measure in the X basis you'll need to use a rotation

$$V\,|\,+\rangle = |\,0\rangle\,, \;\; V\,|\,-\rangle = |\,1\rangle\,,$$

immediately prior to measurement.

Once you have all this, just do the four possible combinations of measurements and see what happens! Use the simulator to verify that the analysis of lecture 6 is correct, and then see how well the real device performs.