Quantum Information 2019
Sheet 11

1. Schmidt decomposition

Show that any pure state of two qubits can be expressed in the form,

$$|\,\psi\rangle_{AB} = \sum_{j\in\{0,1\}} \sqrt{p_j}\,|\,\alpha_j, \beta_j\rangle\,,$$

where $\langle\alpha_i|\alpha_j\rangle = \langle\beta_i|\beta_j\rangle = \delta_{i,j}$ and the $\sqrt{p_j}$ are real and positive.

2. Bell basis measurement

Implement a Bell basis measurement of two qubits. The total measurement (which may be made up of multiple measurements) should leave the two qubits in one of the four Bell states

$$\left|\,\Phi^+\right\rangle = \frac{|\,00\rangle + |\,11\rangle}{\sqrt{2}},\ \left|\,\Phi^-\right\rangle = \frac{|\,00\rangle - |\,11\rangle}{\sqrt{2}},\ \left|\,\Psi^+\right\rangle = \frac{|\,01\rangle + |\,10\rangle}{\sqrt{2}},\ \left|\,\Psi^-\right\rangle = \frac{|\,01\rangle - |\,10\rangle}{\sqrt{2}}.$$

The probability for each of these should be as given by the Born rule when the two qubit state is expressed in this basis.