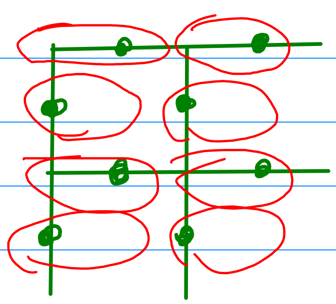
a) There are L vertical lines with L qubits per line. The same for horozontal lines. So the total number of qubits is

$$n = 2L^2$$

b) For every qubit on a vertical edge we can identify plaquette, and vice-versa. Same for qubits on horizontal edges and vertices. So there is the same number of qubits as there are stabilizers



\# plaquettes = \# qubits on vertical lines $= L^2$

Similarly \# vertices $= L^2$

But not all stabilizers are independent. Note that each qubit is acted upon by exactly two plaquette operators, so

$$\prod_p B_p = \mathbb{1} \quad \therefore \quad B_{p'} = \prod_{p \neq p'} B_p$$

Similarly for vertices. So one plaquette operator and one vertex operator can be expressed as a product of the others

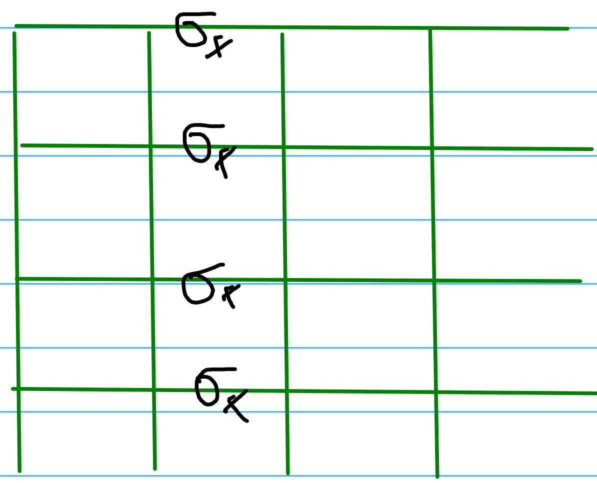
The number of independent plaquette stabilizers is then $L^2 - 1$

And the same for vertices

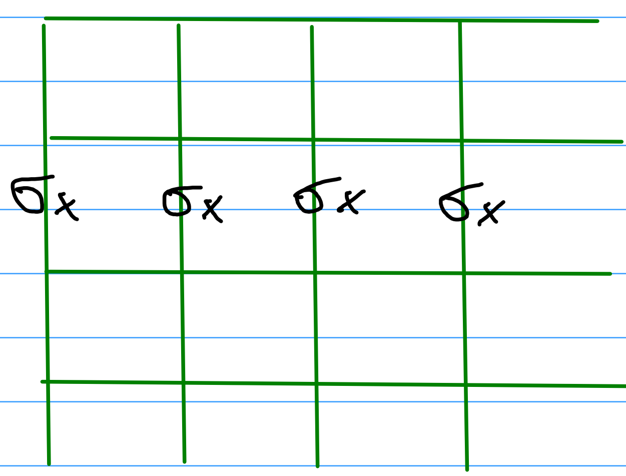
c) $k = \#\,\text{qubits} - \#\,\text{stabilizers}$

$$= 2L^2 - (L^2 - 1) - (L^2 - 1) = 2$$

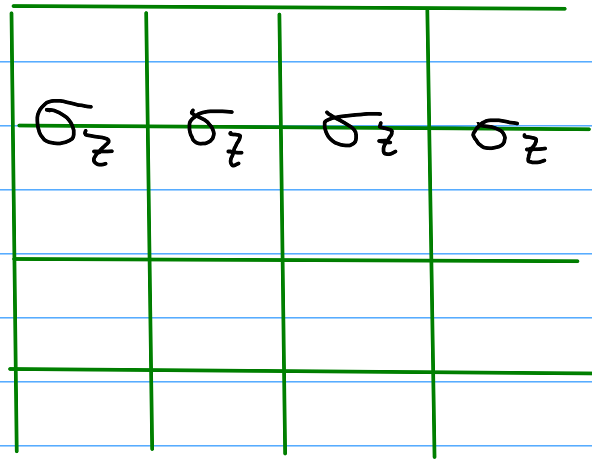
d) $X_1 =$



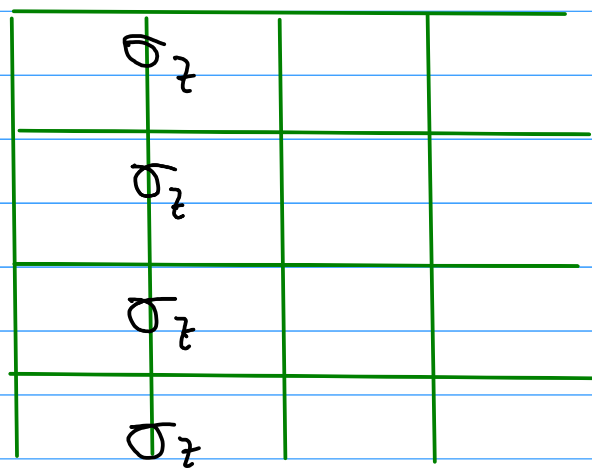
$X_2 =$



$Z_1 =$



$Z_2 =$



2

a) The stabilizers that act on only one shared spin either both act with x or both with z. The stabilizers that act with two shared spins will have one act with $\sigma_x \otimes \sigma_y$ and the other with $\sigma_y \otimes \sigma_z$, and these commute. The only other possibility is acting on no shared spins. So all cases commute.

b) # stabilizers = # spins = $2L_w^2$

How many stablizers are not independent?

For even L_w

$$\prod_{\text{blue } p} W_p = \mathbb{1} \qquad \prod_{\text{white } p} W_p = \mathbb{1}$$

2 are not independent

$\therefore$ $2L_w^2 - 2$ independent stabilizers

$\therefore$ $k = 2$

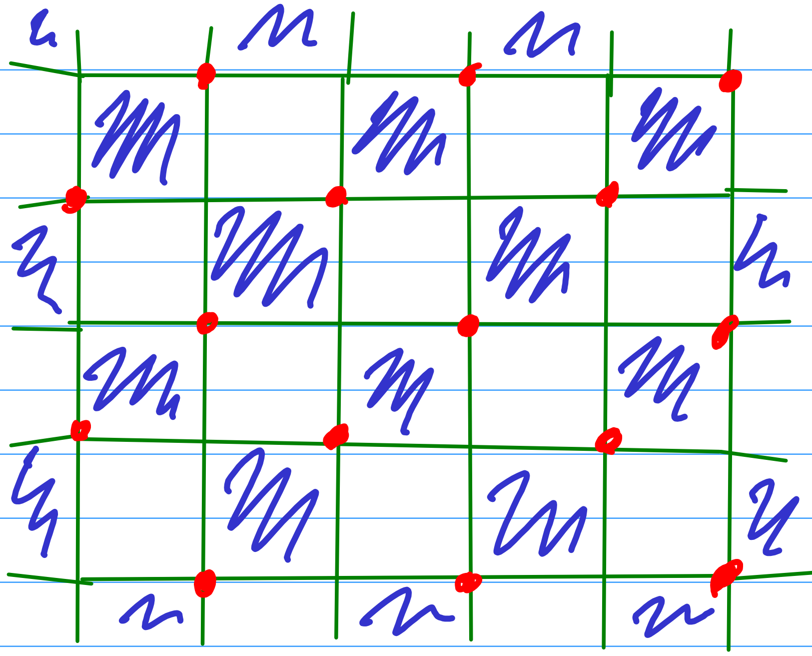
For odd L_w

$\prod_p W_p = \mathbb{1}$ $\therefore$ 1 is not independent

$\therefore$ $2L_w - 1$ independent stabilizers

$\therefore$ $k = 1$

3



$U_j = H$ if j is top right or bottom left of a blue plaquette (red qubits)

$= \mathbb{1}$ otherwise

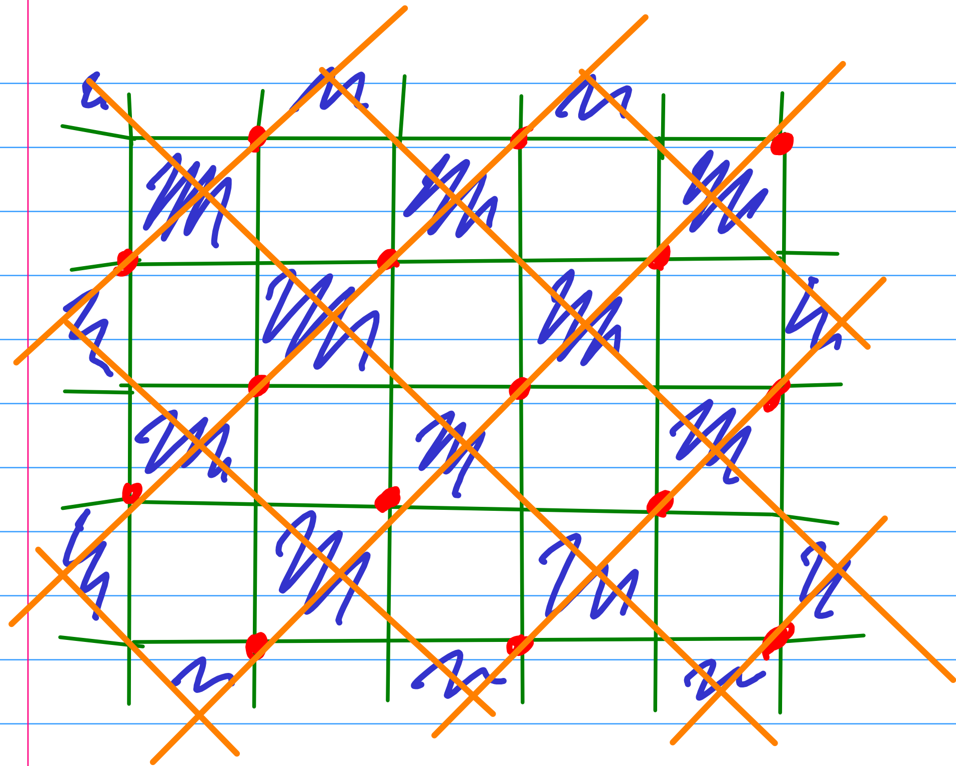
$$U W_p U^\dagger = \sigma_x \sigma_x \sigma_x \sigma_x$$

for blue plaquetts

$$U W_p U^\dagger = \sigma_z \sigma_z \sigma_z \sigma_z$$

for white plaquetts

Same stabilizers as TC



TC lattice shown in orange

white plaquettes → plaquettes

blue plaquettes → vertices