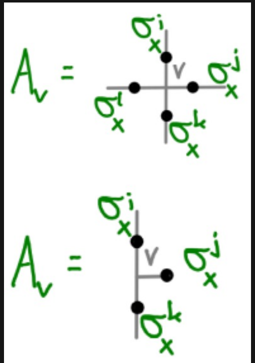
Introduction to LDPC Codes

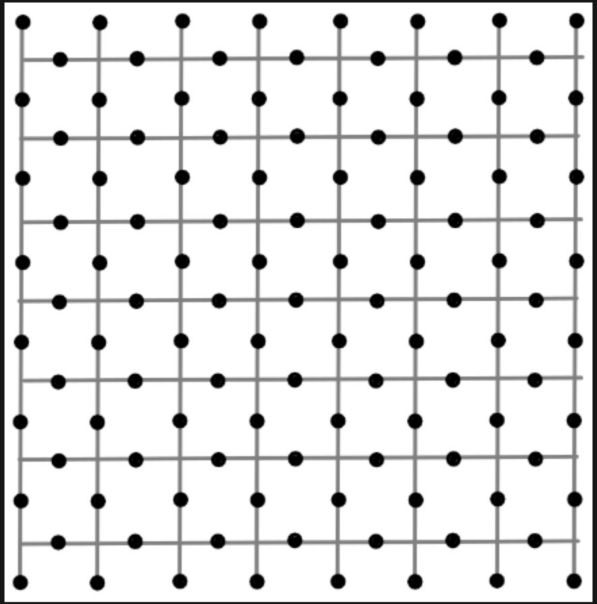
James R. Wootton
IBM Quantum

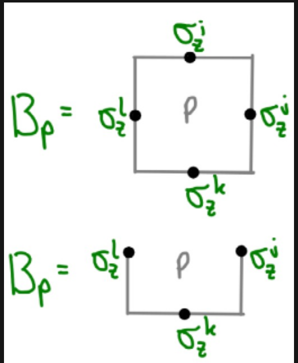
IBM **Quantum**

Why we like the surface code

- Each qubit is involved in only a finite number of syndrome measurements
- Each syndrome measurement requires only a finite number of qubits
- Qubits can be restricted to a 2D lattice with nearest neighbour entangling gates







Why we don’t like the surface code

IBM **Quantum**

- We refer to codes using the parameters [[n,k,d]]
  - n: the number of physical qubits
  - k: the number of logical qubits
  - d: the code distance
- For a surface code

$$n \sim d^2, \qquad k = 1, \qquad d = d$$

- Logical qubits made with the surface code are very expensive

$$R = \lim_{n\to\infty} \frac{k}{n} = 0, \qquad d \sim n^{1/2}$$

- Can we find codes with better scaling, while keeping the nice features?

LDPC codes

- ”Low density parity check” codes are classical EC codes for which
  - Each bit is involved in only a finite number of checks
  - Each check involves only a finite number of bits

- qLDPC codes are the same, but quantum

- Good qLDPC codes are those with good sets of parameters, such as

$$R = \lim_{n\to\infty} \frac{k}{n} = O(1), \qquad d \sim n$$

- But how much do they deviate from a 2D lattice?

qLDPC codes

- We know a few bounds for purely 2D layouts, e.g.
  - $kd^2 \lesssim n$ [Bravyi, Poulin, Terhal 2010]
  - At least $\sqrt{\frac{k}{n}}d$ interactions of range $\sqrt{\frac{k}{\sqrt{d}}}$ are required [Baspin, Krishna 2022]

- These can also be violated, at a price
  - For example

$$k \sim \frac{n}{\log^2 n}, \quad d \sim n^{1/2}$$

but P decays only superpolynomially

Hierarchical memories: Simulating quantum LDPC codes with local gates

Christopher A. Pattison[1], Anirudh Krishna[2,3], and John Preskill[1,4]

[1] *Institute for Quantum Information and Matter, California Institute of Technology, Pasadena, CA 91125*
[2] *Department of Computer Science, Stanford University, Stanford, CA, 94305*
[3] *Stanford Institute for Theoretical Physics, Stanford University, Stanford, CA, 94305*
[4] *AWS Center for Quantum Computing, Pasadena CA 91125*

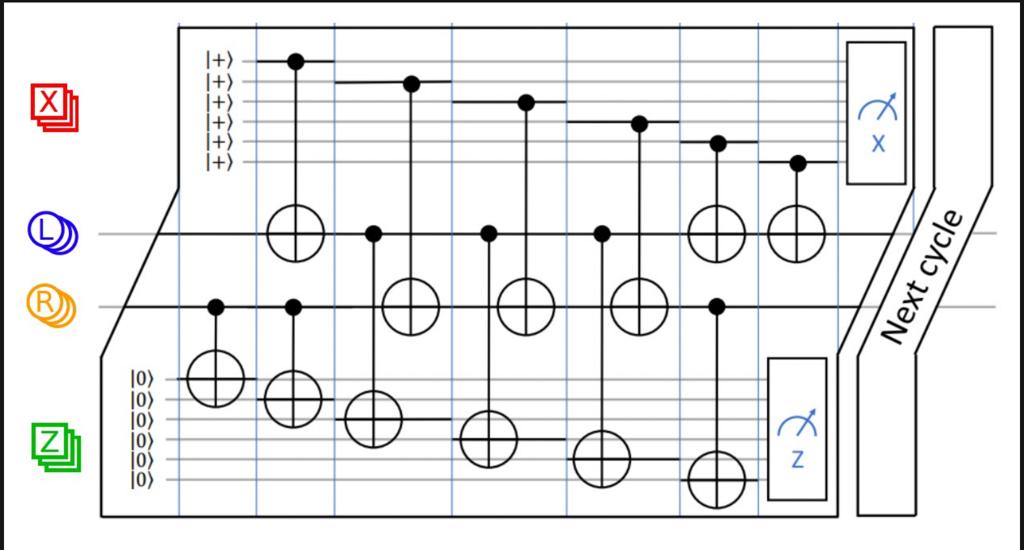
March 9, 2023

Abstract

Constant-rate low-density parity-check (LDPC) codes are promising candidates for constructing efficient fault-tolerant quantum memories. However, if physical gates are subject to geometric-locality constraints, it becomes challenging to realize these codes. In this paper, we construct a new family of $[\![N, K, D]\!]$ codes, referred to as hierarchical codes, that encode a number of logical qubits $K = \Omega(N/\log(N)^2)$. The N^{th} element $\mathcal{H}_N$ of this code family is obtained by concatenating a constant-rate quantum LDPC code with a surface code; nearest-neighbor gates in two dimensions are sufficient to implement the syndrome-extraction circuit $C_N^{\mathcal{H}}$ and achieve a threshold. Below threshold the logical failure rate vanishes superpolynomially as a function of the distance $D(N)$. We present a bilayer architecture for implementing $C_N^{\mathcal{H}}$, and estimate the logical failure rate for this architecture. Under conservative assumptions, we find that the hierarchical code outperforms the basic encoding where all logical qubits are encoded in the surface code.

qLDPC codes at IBM

- At IBM we want codes with
  - High distance and encoding rate
  - A high threshold (or pseudothreshold) for circuit noise
  - Superconducting qubit implementation
  - A short-depth syndrome extraction circuit



qLDPC codes at IBM

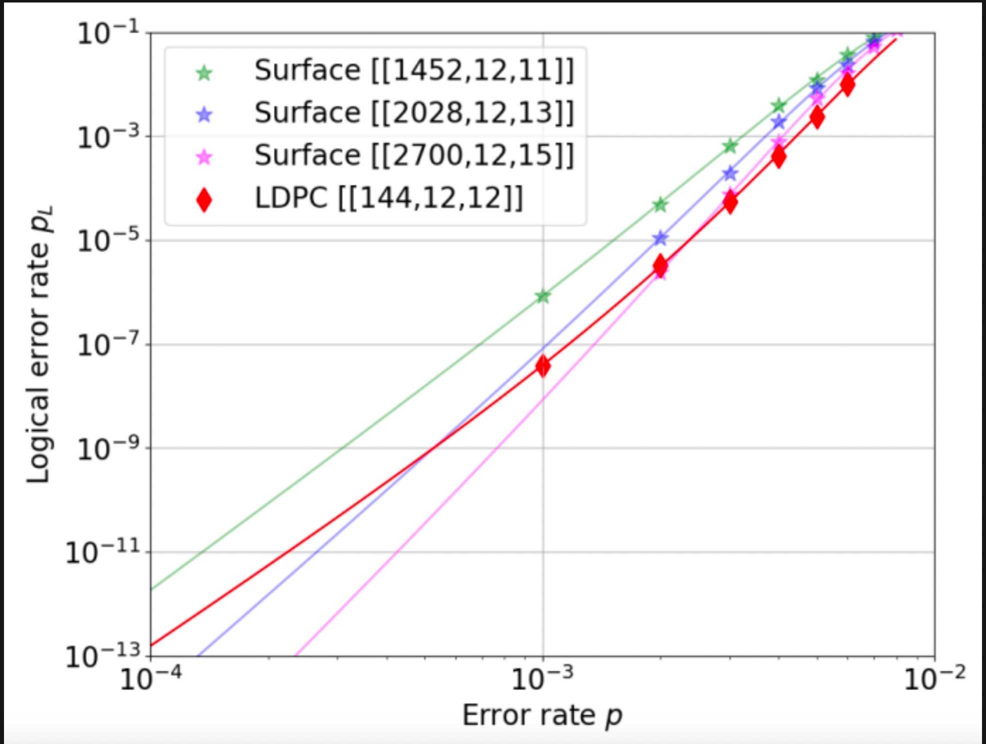
- Answer comes from ”bivariant bicycle codes”
  - Variant of quasi-cyclic codes [Kovalev, Pryadko 2013]

$[[n,k,d]]$	Net Encoding Rate r	ℓ, m	A	B
$[[72,12,6]]$	$1/12$	$6,6$	x^3+y+y^2	y^3+x+x^2
$[[90,8,10]]$	$1/23$	$15,3$	x^9+y+y^2	$1+x^2+x^7$
$[[108,8,10]]$	$1/27$	$9,6$	x^3+y+y^2	y^3+x+x^2
$[[144,12,12]]$	$1/24$	$12,6$	x^3+y+y^2	y^3+x+x^2
$[[288,12,18]]$	$1/48$	$12,12$	$x^3+y^2+y^7$	y^3+x+x^2
$[[360,12,\leq 24]]$	$1/60$	$30,6$	x^9+y+y^2	$y^3+x^{25}+x^{26}$
$[[756,16,\leq 34]]$	$1/95$	$21,18$	$x^3+y^{10}+y^{17}$	$y^5+x^3+x^{19}$

Compare to [[2028,12,13]] surface code: $r = 1/169$

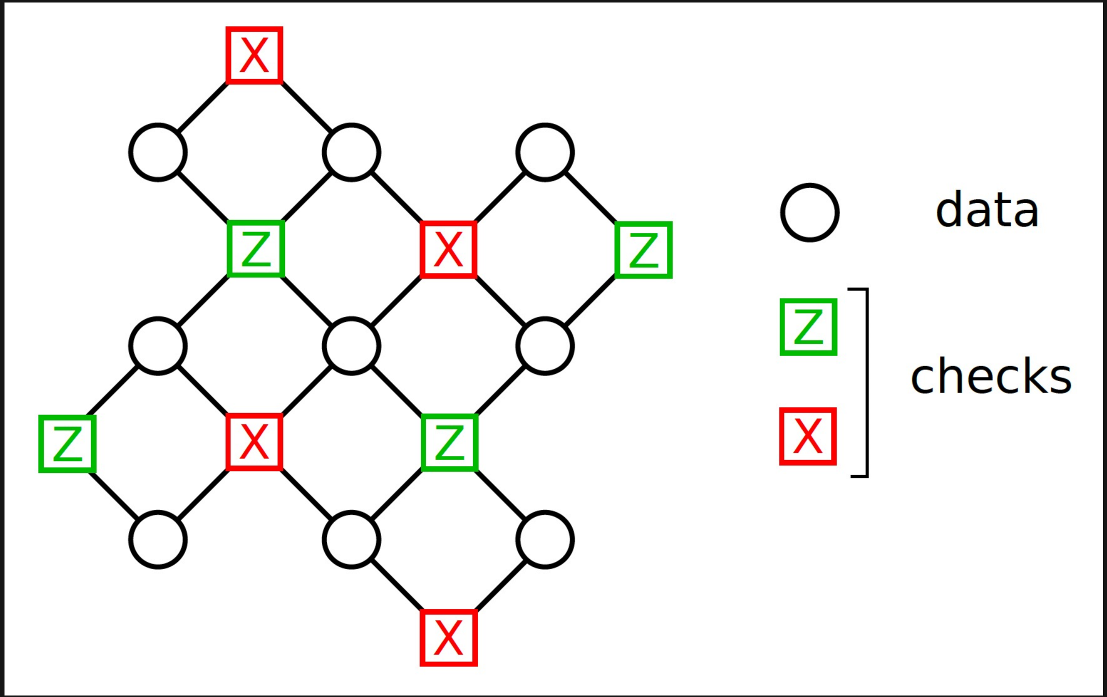
qLDPC codes at IBM

– Matches surface code performance, but with 10x fewer qubits!



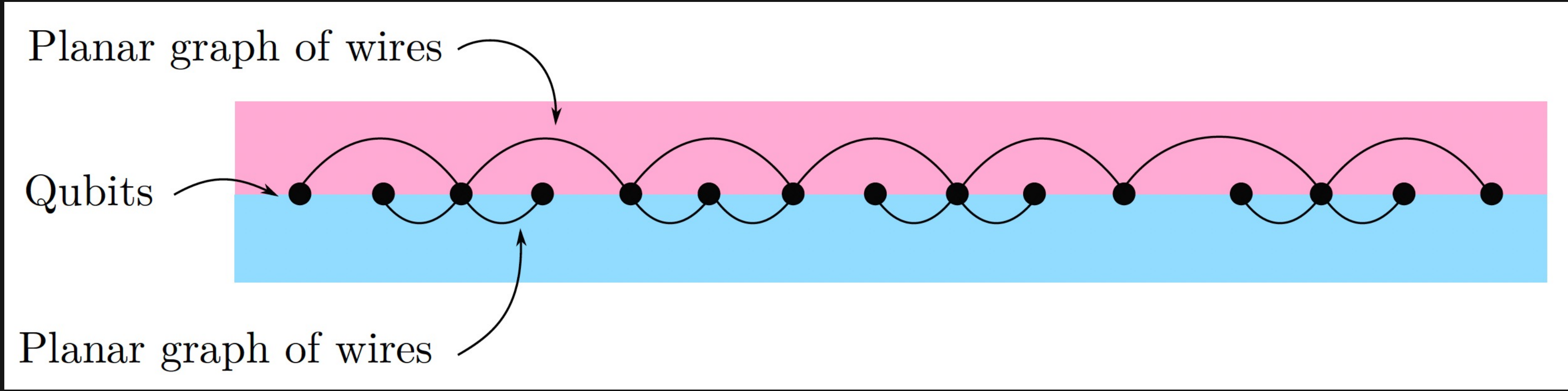
Bilayer representation

– First: Tanner graphs



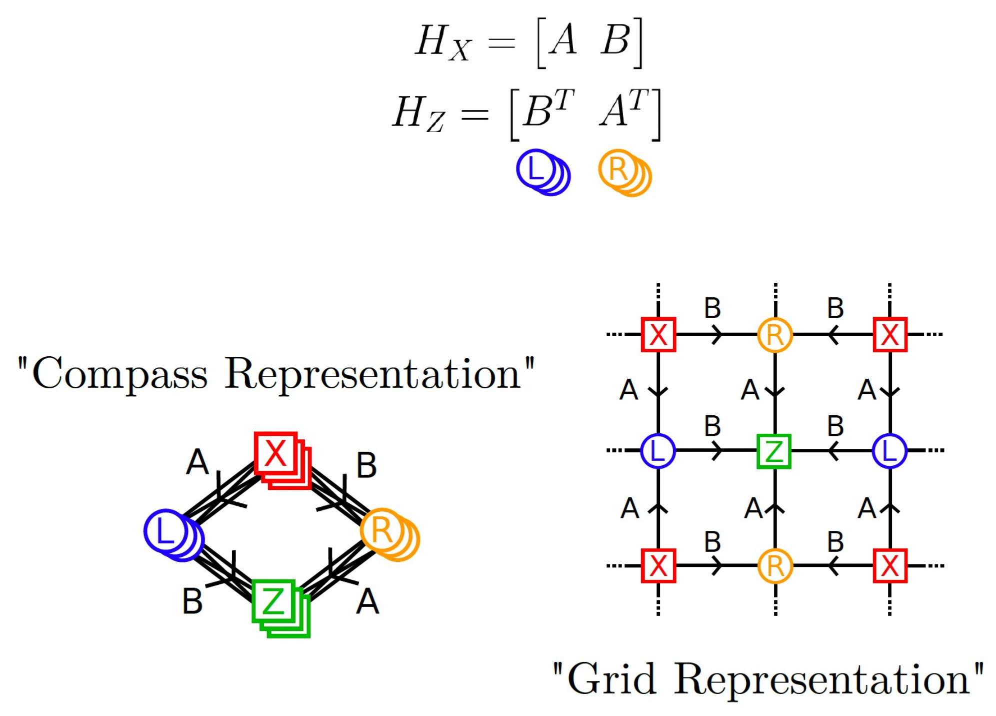
Bilayer representation

- If planar graphs aren’t good enough, we go for thickness-2
  - Union of two planar graphs



Bilayer representation

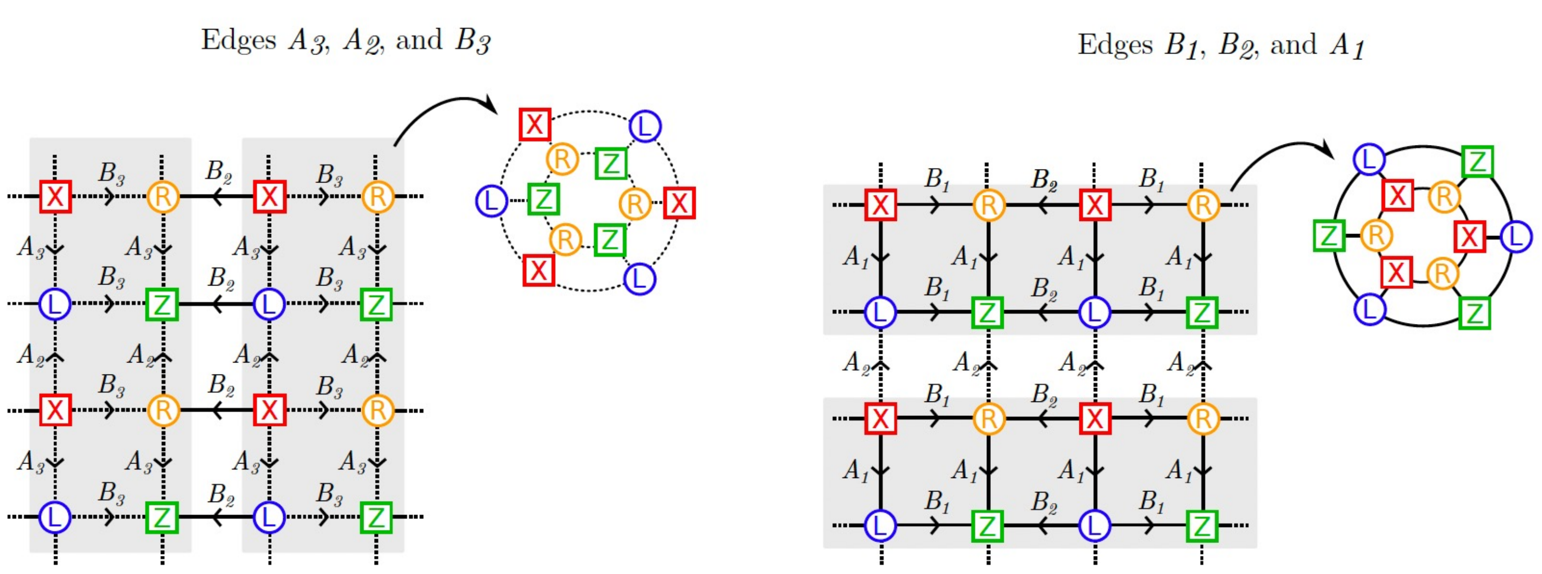
– Tanner graph for quasi-cyclic codes



Bilayer representation

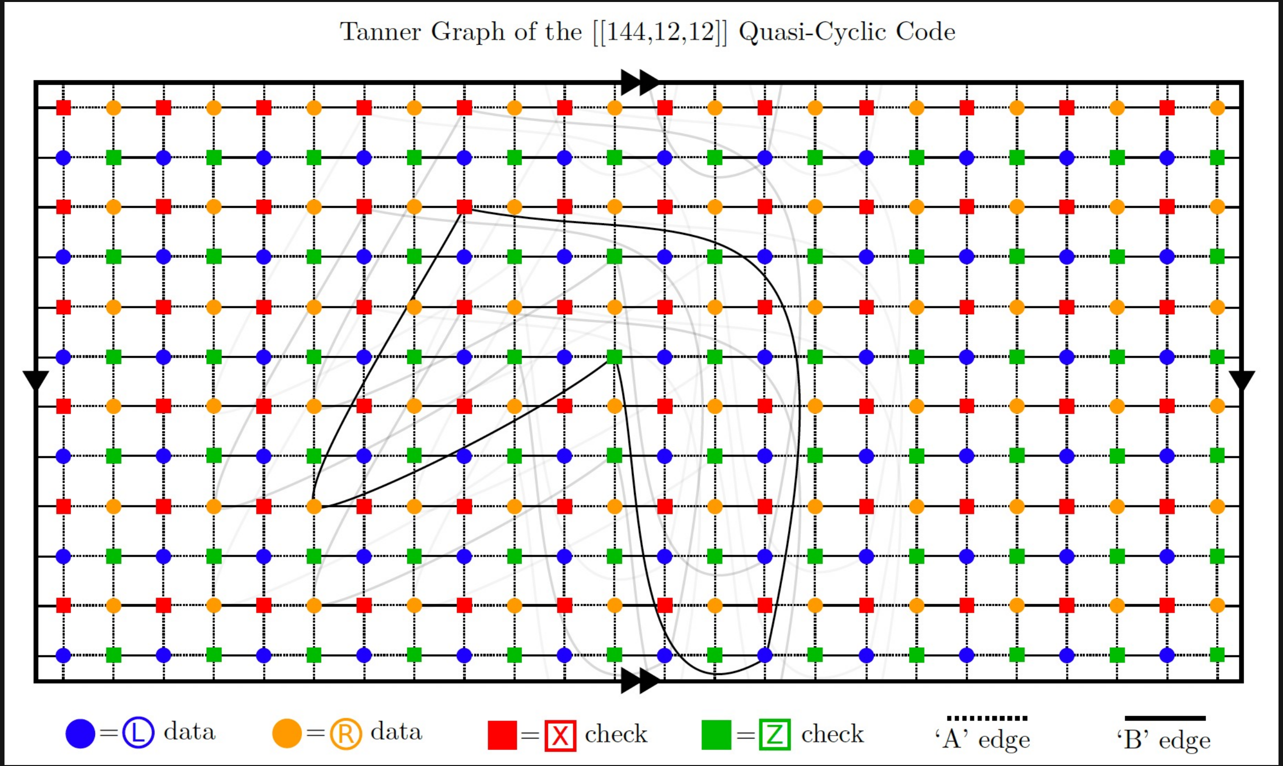
– Visual proof of thickness-2

$$A = A_1 + A_2 + A_3 \qquad B = B_1 + B_2 + B_3$$

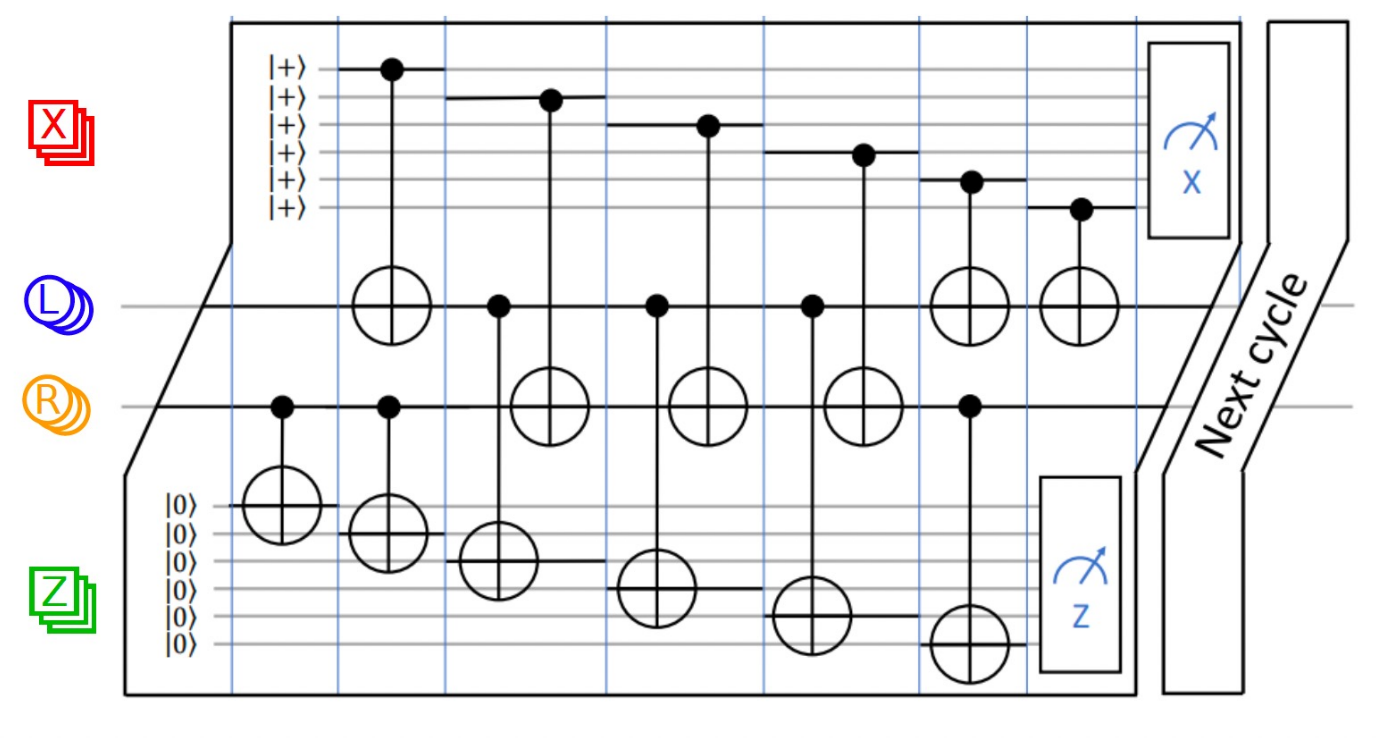


Bilayer representation

– Tanner graph of [[114,12,12]]



Syndrome measurement circuit



Pseudo-thresholds around 0.8%

Conclusions

- qLDPC codes that outperform the surface code
  - Better rate
  - Same error suppression
  - Similar pseudo-threshold
- The cost is a more complex Tanner graph
  - But bilayer architecture is something we can achieve!

Bravyi et al., arXiv:2308.07915 (2023)

Thanks for your attention

qisk.it/decoders