Quantum Computation and Error Correction: Exercise Sheet 4

December 2, 2025

Problem 1. Calibration (or confusion) matrix:

- **Problem 1.1.** (4 marks) Given this dataset which is given in a dictionary of

  $$\text{expected bitstring} : [\text{list of experimental results}]$$

  for one qubit it may look like

  $$'0' : [0, 1, 0, 1, 0, 0, 0, 0 \dots]$$
  $$'1' : [1, 0, 0, 1, 1, 1, 0, 1 \dots]$$

  and for two

  $$'00' : [00, 01, 10, 00, 00, 01, 10, 01 \dots]$$
  $$'11' : [10, 00, 01, 11, 10, 01, 00, 01 \dots]$$

  Build the calibration/confusion matrix programmatically (preferably). The data will be for a 3 qubit experiment and contain 10,000 shots per prepared state (from 000 to 111). Do so for the high and low noise data.

- **Problem 1.2.** (3 marks) **Determine** the approximate flipping probability of each bit in both low and high noise regimes.

- **Problem 1.3.** (3 marks) Is the resulting correction matrix (inverse of the calibration/confusion matrix) on an arbitrary result probability preserving in either cases? If it is not, what do you think could minimize the error?

Problem 2. Zero noise extrapolation:

- **Problem 2.1.** (8 marks) Given the following global depolarizing error model on n qubits

  $$E(\rho) = (1-p)\rho + p\frac{I}{2^n} \tag{1}$$

  perform *Richardson extrapolation* on two and three non-zero noise probabilities p_1, p_2, p_3 where $0 < p_1 < p_2 < p_3 < 1$ to try and **find** the limit at which $p = 0$.

- **Problem 2.2.** (2 marks) What happens as p_1 increases? Remember that at the limit of p=1, of course you cannot really have 'noisier' noise levels.

  Hint: The effect of the above channel on an observable of interest O gives $\hat{\mu} = (1-p)\text{Tr}(\rho O) + p\frac{\text{Tr}(O)}{2^n}$. The 'hat' here represents the noisy version of μ, which ideally would be $\text{Tr}(\rho O)$. You can look at the appendix of this paper (link) for some more help with the derivation.