Quantum Information 2019
Exercise 6

1. Single Qubit Tomography

Prepare four different single qubit states using Qiskit with a noisy simulation (or real device).

For all states, measure in the X, Y and Z bases and use the results to calculate the expectation values $\langle X \rangle$, $\langle Y \rangle$ and $\langle Z \rangle$. With these, construct the corresponding density matrix for each of the four states using the relation,

$$\rho = \frac{1 + \langle X \rangle X + \langle Y \rangle Y + \langle Z \rangle Z}{2}. \tag{1}$$

(a) Check whether the density operators are indeed $\mathrm{Tr}(\rho_i) = 1$ and Hermitian and positive, as required.

(b) Diagonalize the density operators. Comment on their similarities and differences with the intended states $|0\rangle$, $|1\rangle$, $|+\rangle$ and $|-\rangle$.

2. Kraus operators

a) Consider single qubit noise that applies a σ_x with probability p_x, a σ_y with probability p_y and a σ_z with probability p_z. The probability that nothing is applied is $1 - p_x - p_y - p_z$. What are the Kraus operators for this?

b) The single qubit noise operator

$$\varepsilon(\rho) = (1-p)\rho + p\,\frac{1}{2}\sigma_0$$

can be expressed in the same form as part (a) with suitably chosen p_x, p_y and p_z. Find these.

c) Consider the following gate that interacts two qubits,

$$H = \frac{1}{2}\left(\sigma_0 \otimes \sigma_0 + \sigma_x \otimes \sigma_0 + \sigma_0 \otimes \sigma_z - \sigma_x \otimes \sigma_z\right). \tag{2}$$

If initial state of the second qubit is $|+\rangle$, what are the Kraus operators for the resulting process on the first qubit after a time t.

d) As (c), but with initial state $|0\rangle$ for the second qubit.